

# What Drives Anomaly Returns?

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## Abstract

We decompose the returns of five well-known anomalies into cash flow and discount rate news. Common patterns emerge across all factor portfolios and their mean-variance efficient combination. The main source of anomaly return variation is news about cash flows. Anomaly cash flow and discount rate components are strongly negatively correlated, and this negative correlation is driven by news about long-run cash flows. Interestingly, anomaly cash flow (discount rate) news is approximately uncorrelated with market cash flow (discount rate) news. These rich empirical patterns are useful for guiding specifications of asset pricing models and evaluating myriad theories of anomalies.

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# 1 Introduction

Researchers in the past 30 years have uncovered robust patterns in stock returns that contradict classic asset pricing theories. A prominent example is that value stocks outperform growth stocks, even though these stocks are similarly exposed to fluctuations in the overall stock market. To exploit such anomalies, investors can form long-short portfolios (e.g., long value and short growth) with high average returns and near-zero market risk. These long-short anomaly portfolios form an important part of the mean-variance efficient (MVE) portfolio and thus the stochastic discount factor (SDF) that prices all assets. For instance, in the five-factor Fama and French (2015) model non-market factors account for 85% of the variance in the model’s implied SDF.<sup>1</sup>

Researchers sharply disagree about the source of these non-market factors. Several different risk-based and behavioral models can explain why long-short portfolios based on valuation ratios and other characteristics earn high average returns.<sup>2</sup> In this paper, we introduce an efficient empirical technique for decomposing anomaly portfolio returns, as well as their mean-variance efficient combination, into cash flow and discount rate shocks as in Campbell (1991). These decompositions provide useful new facts that guide the specification of asset pricing theories.

To see how this decomposition relates to theories, consider at one extreme the model of noise trader risk proposed by De Long et al. (1990). In this model, firm dividends (cash flows) are constant, implying that all return variation arises from changes in discount rates. At the other extreme, consider the simplest form of the Capital Asset Pricing Model (CAPM) in which firm betas and the market risk premium are constant. In this setting, expected returns (discount rates) are constant, implying that all return variation arises from changes in expected cash flows. More generally, applying our empirical methodology to simulated

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<sup>1</sup>Using data from 1963 to 2017, a regression of the mean-variance efficient combination of the five Fama-French factors on the market factor yields an  $R^2$  of 15%.

<sup>2</sup>Risk-based models include Berk, Green, and Naik (1999), Zhang (2005), Lettau and Wachter (2007), and Kogan and Papanikolaou (2013). Behavioral models include Barberis, Shleifer, and Vishny (1998), Hong and Stein (1999), and Daniel, Hirshleifer, and Subrahmanyam (2001).

data from any risk-based or behavioral theory provides a novel test of whether the model matches the empirical properties of cash flow and discount rate shocks to anomaly portfolios and their MVE combination.

Our empirical work focuses on five well-known anomalies—value, size, profitability, investment, and momentum—and yields three sets of novel findings for theories to explain. First, for all five anomalies, cash flows explain more variation in anomaly returns than do discount rates. Second, for all five anomalies, shocks to cash flows and discount rates are strongly negatively correlated. This correlation is driven by shocks to long-run cash flows, as opposed to shocks to short-run (one-year) cash flows. That is, firms with negative news about long-run cash flows tend to experience persistent increases in discount rates. This association contributes significantly to return variance in anomaly portfolios. Third, for all five anomalies, anomaly cash flow and discount rate components exhibit weak correlations with market cash flow and discount rate components. In fact, when we combine all five anomalies into an MVE portfolio, discount rate shocks to this anomaly MVE portfolio are slightly negatively correlated with market discount rate shocks. This fact is surprising if one interprets discount rates as proxies for investor risk aversion as it suggests that increased aversion to market risk is, if anything, associated with decreased aversion to anomaly risks. Furthermore, cash flow shocks to the market are uncorrelated with cash flow shocks to the anomaly MVE portfolio, indicating that the two portfolios are exposed to distinct fundamental risks.

These findings cast doubt on three types of theories of anomalies. First, theories in which discount rate variation is the primary source of anomaly returns, such as De Long et al. (1990), are inconsistent with the evidence on the importance of cash flow variation. The main reason that anomaly portfolios are volatile is that cash flow shocks are highly correlated across firms with similar characteristics. For example, the long-short investment portfolio is volatile mainly because the cash flows of a typical high-investment firm are more strongly correlated with the cash flows of other high-investment firms than with those of low-investment firms. Second, theories that emphasize commonality in discount rates,

such as theories of time-varying risk aversion and those of common investor sentiment, are inconsistent with the low correlations between discount rate shocks to anomaly returns and those to market returns. Third, theories in which anomaly cash flow shocks are strongly correlated with market cash flow shocks—i.e., cash flow beta stories—are inconsistent with empirical correlations that are close to zero.

In contrast, some theories of firm-specific biases in information processing and theories of firm-specific changes in risk are consistent with our three main findings. Such theories include behavioral models in which investors overextrapolate news about firms' long-run cash flows (e.g., Daniel, Hirshleifer, and Subrahmanyam (2001)) and rational models in which firm risk increases after negative news about long-run cash flows (e.g., Kogan and Papanikolaou (2013)). In these theories, discount rate shocks amplify the effect of cash flow shocks on returns, consistent with the robustly negative empirical correlation between these shocks. These theories are also consistent with low correlations between anomaly return components and market return components.

We further search for commonality by relating components of anomaly and market returns to measures of macroeconomic fluctuations, including changes in proxies for risk aversion, investor sentiment, and intermediary leverage. Cash flow shocks to the anomaly MVE portfolio are significantly positively correlated with shocks to broker-dealer leverage but uncorrelated with other macroeconomic measures. Although market cash flow shocks are also positively correlated with broker-dealer leverage, market cash flows are significantly positively correlated with key macroeconomic aggregates, such as consumption and income growth and the labor share, and negatively correlated shocks with a measure of aggregate risk aversion. Discount rate shocks to the anomaly MVE portfolio are negatively correlated with shocks to broker-dealer leverage, consistent with models of limited intermediary capital, and shocks to the Baker and Wurgler (2006) sentiment index. We find little evidence that anomaly cash flows or discount rates are related to consumption or income growth, or measures of aggregate risk aversion.

Our approach builds on the present-value decomposition of Campbell and Shiller (1988) and Campbell (1991) that Vuolteenaho (2002) applies to individual firms. We directly estimate firms' discount rate shocks using an unbalanced panel vector autoregression (VAR) in which we impose the present-value relation to derive cash flow shocks. Different from prior work, we analyze the implications of our firm-level estimates for priced (anomaly) factor portfolios to investigate the fundamental drivers of these factors' returns. The panel VAR, as opposed to a time-series VAR for each anomaly portfolio, fully exploits information about the cross-sectional relation between shocks to characteristics and returns. Our panel approach allows us to consider more return predictors, substantially increases the precision of the return decomposition, and mitigates small-sample issues.<sup>3</sup> Motivated by Chen and Zhao's (2009) finding that VAR results can be sensitive to variable selection, we show that our return decompositions are robust across many different specifications.

Vuolteenaho (2002) finds that, at the firm-level, cash flows are the main drivers of returns, which we confirm in our sample. He further argues that, at the market level, discount rates are the main drivers of returns. Cohen, Polk, and Vuolteenaho (2003), Cohen, Polk, and Vuolteenaho (2009), and Campbell, Polk, and Vuolteenaho (2010) use various approaches to argue that cash flows are the main drivers of risks and expected returns of the long-short value-minus-growth portfolio, broadly consistent with our finding for value. Our study is unique in that we analyze multiple anomalies (not just value), along with the market and, most importantly, the mean-variance efficient portfolio. This joint analysis uncovers robust patterns across anomalies and the MVE portfolio.

Fama and French (1995) document that changes in earnings-to-price ratios for their HML and SMB portfolios exhibit a factor structure, consistent with our findings. However, we examine cash flow shocks extracted using a present value equation in which many charac-

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<sup>3</sup>More subtly, inferring cash flow and discount rate shocks directly from a VAR estimated using returns and cash flows of rebalanced anomaly portfolios (trading strategies) obfuscates the underlying sources of anomaly returns. Firms' weights in anomaly portfolios can change dramatically with the realizations of stock returns and firm characteristics. In Internet Appendix A, we provide extreme examples in which, for example, firms' expected cash flows are constant but direct VAR estimation suggests that all return variation in the rebalanced anomaly portfolio comes from cash flow shocks.

teristics predict earnings at various horizons. Unlike Fama and French (1995), we find a strong relation between the factor structure in cash flow shocks and the factor structure in returns. They acknowledge their failure to find this relation as the “weak link” in their story and “speculate that this negative result is caused by noise in [their] measure of shocks to expected earnings.” Using the present value relation also allows us to analyze discount rates. In addition, our analysis includes the investment, profitability, and momentum anomalies.

Lyle and Wang (2015) estimate the discount rate and cash flow components of firms’ book-to-market ratios by forecasting one-year returns using return on equity and book-to-market ratios. They focus on stock return predictability at the firm level and do not analyze the sources of anomaly returns. In subsequent work, Haddad, Kozak, and Santosh (2017) propose a principal components approach to forecasting portfolio returns. Consistent with our results, they find low correlation between market discount rates and long-short factor portfolios’ discount rates. Our work is also related to studies that use the log-linear approximation of Campbell and Shiller (1988) for price-dividend ratios, typically applied to the market portfolio (see Campbell (1991), Larrain and Yogo (2008), van Binsbergen and Koijen (2010), and Kelly and Pruitt (2013)).

The paper proceeds as follows. Section 2 provides examples of theories’ implications for anomaly cash flows and discount rates. Section 3 introduces the empirical model. Section 4 describes the data and empirical specifications. Section 5 discusses the VAR estimation, while Section 6 presents firm- and portfolio-level results. Section 7 shows robustness tests, and Section 8 concludes.

## 2 Theory

Empirical research identifies several asset pricing anomalies in which firm characteristics, such as firm profitability and investment, predict firms’ stock returns even after controlling for market beta. Modern empirical asset pricing models therefore postulate multiple factors (e.g., Fama and French (1993, 2015), Carhart (1997)), including non-market factors defined

as long-short portfolios sorted on such characteristics.

In this paper, we decompose the returns to long-short anomaly portfolios and their mean-variance efficient (MVE) combination into updates in expectations of current and future cash flows, cash flow ( $CF$ ) news, and updates in expectations of future returns, discount rate ( $DR$ ) news. For an arbitrary factor portfolio  $F_k$ , this decomposition yields:

$$R_{F_k,t} - E_{t-1} [R_{F_k,t}] = CF_{F_k,t} - DR_{F_k,t}. \quad (1)$$

Decomposing the returns of long-short anomaly portfolios and their MVE combination is useful as it provides additional moments that can guide specifications of asset pricing models. Long-short anomaly portfolio returns are volatile and have market betas that are usually close to zero. Thus, the firms in such portfolios must be subject to correlated shocks. As we will explain, theories of anomaly returns that feature a meaningful cross section of firms have important implications for whether these shocks are correlated cash flow or discount rate shocks. The mean-variance efficient combination of these factors is also of interest as shocks to this portfolio's return are proportional to shocks to the stochastic discount factor  $M_t$  (e.g., Roll (1976)):

$$M_t - E_{t-1} [M_t] = b (R_{MVE,t} - E_{t-1} [R_{MVE,t}]), \quad (2)$$

where  $R_{MVE,t} = \sum_{k=1}^K \omega_k R_{F_k,t}$  is the return to the MVE portfolio at time  $t$ , expressed as a linear function of  $K$  factor returns, and where  $b < 0$ . Thus, the risks driving marginal utility of the marginal agent are reflected in, or indeed arise from, correlated shocks to this portfolio. Understanding the nature and magnitudes of cash flow and discount rate shocks to the MVE portfolio is therefore informative for all asset pricing models.

## 2.1 The Return Decomposition

Recall from Campbell (1991) that we can decompose shocks to log stock returns into shocks to expectations of cash flows and returns:<sup>4</sup>

$$r_{i,t+1} - E_t[r_{i,t+1}] \approx CF_{i,t+1} - DR_{i,t+1}, \quad (3)$$

where

$$CF_{i,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa^{j-1} \Delta d_{i,t+j}, \quad (4)$$

$$DR_{i,t+1} = (E_{t+1} - E_t) \sum_{j=2}^{\infty} \kappa^{j-1} r_{i,t+j}, \quad (5)$$

and where  $\Delta d_{i,t+j}(r_{i,t+j})$  is the log of dividend growth (log of gross return) of firm  $i$  from time  $t + j - 1$  to time  $t + j$ , and  $\kappa$  is a log-linearization constant slightly less than 1.<sup>5</sup> In words, return innovations are due to updates in beliefs about current and future dividend growth or future expected returns.

We define anomaly returns as the value-weighted returns of stocks ranked in the highest quintile of a given priced characteristic minus the value-weighted returns of stocks ranked in the lowest quintile. We define anomaly cash flow news as the cash flow news for the top quintile portfolio minus the news for the bottom quintile portfolio. We similarly define anomaly discount rate shocks. In the empirical section, we describe our method in detail. Next we discuss the implications of this decomposition of anomaly and MVE portfolio returns for specific models of the cross-section of stock returns.

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<sup>4</sup>The operator  $(E_{t+1} - E_t)x$  represents  $E_{t+1}[x] - E_t[x]$ : the update in the expected value of  $x$  from time  $t$  to time  $t + 1$ . The equation relies on a log-linear approximation of the price-dividend ratio around its sample average.

<sup>5</sup>A similar decomposition holds for non-dividend paying firms, assuming clean-surplus earnings (see, Ohlson (1995), and Vuolteenaho (2002)). In this case, the relevant cash flow is the log of gross return on equity. The discount rate shock takes the same form as in Equation (5).



## 2.2 Relating the Decomposition to Anomalies

Theories of anomalies propose that the properties of investor beliefs and firm cash flows vary with firm characteristics. The well-known value premium provides a useful illustration. De Long et al. (1990) and Barberis, Shleifer and Vishny (1998) are examples of behavioral models that could explain this anomaly, while Zhang (2005) and Lettau and Wachter (2007) are examples of rational explanations.

First, consider a multi-firm generalization of the De Long et al. (1990) model of noise trader risk. In this model, firm cash flows are constant but stock prices fluctuate because of random demand from noise traders, driving changes in firm book-to-market ratios. As expectations in Equation (4) are rational, there are no cash flow shocks in this model. By Equation (3), all shocks to returns are due to discount rate shocks. The constant cash flow assumption is clearly stylized. However, if one in the spirit of this model assumes that value and growth firms have similar cash flow exposures, the variance of net cash flow shocks to the long-short portfolio would be small relative to the variance of discount rate shocks. Thus, an empirical finding that discount rate shocks only explain a small fraction of return variance to the long-short value portfolio would be inconsistent with this model.

Barberis, Shleifer, and Vishny (BSV, 1998) propose a model in which investors overextrapolate from long sequences of past firm earnings when forecasting future firm earnings. Thus, a firm that repeatedly experiences low earnings will be underpriced (a value firm) as investors are too pessimistic about its future earnings. The firm will have high expected returns as future earnings on average are better than investors expect. Growth firms will have low expected returns for analogous reasons. In this model, cash flow and discount rate shocks are intimately linked. Negative shocks to cash flows cause investors to expect low expected future cash flows. But these irrationally low expectations manifest as positive discount rate shocks in Equations (4) and (5), as the econometrician's expectations are rational. Thus, this theory predicts a strong negative correlation between cash flow and discount rate shocks at the firm and anomaly levels.

Zhang (2005) provides a rational explanation for the value premium based on a model of firm production. Persistent idiosyncratic productivity (earnings) shocks by chance make firms into either value or growth firms. Value firms, which have low productivity, have more capital than optimal because of adjustment costs. These firms' values are very sensitive to negative aggregate productivity shocks as they have little ability to smooth such shocks through disinvesting. Growth firms, on the other hand, have high productivity and suboptimally low capital stocks and therefore are not as exposed to negative aggregate shocks. Value (growth) firms' high (low) betas with respect to aggregate shocks justify their high (low) expected returns. Similar to BSV, this model predicts a negative relation between firm cash flow and discount rate shocks. Different from BSV, the model predicts that the value anomaly portfolio has cash flow shocks that are positively related to market cash flow shocks because value stocks are more sensitive to aggregate technology shocks than growth stocks.

Lettau and Wachter (2007) propose a duration-based explanation of the value premium. In their model, growth firms are, relative to value firms, more exposed to shocks to market discount rates and long-run cash flows, which are not priced, and less exposed to shocks to short-run market cash flows, which are priced. This model implies that short-run cash flow shocks to the long-short value portfolio are positively correlated with short-run market cash flow shocks. In addition, discount rate and long-run cash flow shocks to the value portfolio are negatively correlated with market discount rate and long-run cash flow shocks, respectively.

### **2.3 Relating the Decomposition to the Stochastic Discount Factor**

Prior studies (e.g., Campbell (1991) and Cochrane (2011)) decompose market returns into cash flow and discount rate news. They argue that the substantial variance of market discount rate news has deep implications for the joint dynamics of investor preferences and aggregate cash flows in asset pricing models. For instance, the Campbell and Cochrane (1999) model relies on strong time-variation in investor risk aversion—i.e., the price of risk—which

is consistent with the high variance of market discount rate news.

The modern consensus is that the mean-variance efficient (MVE) portfolio and thus the stochastic discount factor (SDF) includes factors other than the market. By the logic above, decomposing MVE portfolio returns into cash flow and discount rate news also can inform specifications of asset pricing models. For example, the Campbell and Cochrane (1999) model's large time-variation in investor risk aversion implies an important role for discount rate shocks and a common component in the discount rate shocks across the factor portfolios in the SDF.

All models that feature a cross-section of stocks have implications for the return decomposition of anomaly portfolios and the MVE portfolio. As an example, Kogan and Papanikolaou (2013) propose a model in which aggregate investment-specific shocks, uncorrelated with market productivity shocks that affect all capital, have a negative price of risk. Value and growth firms have similar exposure to market productivity shocks, but growth firms have higher exposure to the investment-specific shock. These two aggregate cash flow shocks are the primary drivers of returns to the MVE portfolio in their economy. However, since book-to-market ratios increase with discount rates, discount rate shocks are also present and there is a negative correlation between cash flow and discount rate shocks.

## 2.4 The Empirical Model

Most theories of anomalies, including those above, apply to individual firms. To test these theories, one must analyze firm-level cash flow and discount rate news and then aggregate these shocks into anomaly portfolio news. As we explain in Internet Appendix A, extracting cash flow and discount rate news directly from rebalanced portfolios, such as the Fama-French value and growth portfolios, can lead to mistaken inferences as trading itself confounds the underlying firms' cash flow and discount rate shocks.<sup>6</sup> We assume that firm-level expected

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<sup>6</sup>In Internet Appendix A, we provide an example of a value-based trading strategy. The underlying firms only experience discount rate shocks, but the traded portfolio is driven solely by cash flow shocks as a result of rebalancing.

log returns are linear in observable variables ( $X$ ):

$$E_t[r_{i,t+1}] = \delta_0 + \delta_1' X_{it} + \delta_2' X_{At}. \quad (6)$$

Here,  $X_{it}$  is a vector of firm-specific characteristics, such as book-to-market and profitability, and  $X_{At}$  is a vector of aggregate variables, such as the risk-free rate and aggregate book-to-market ratio, all measured in logs. Define the  $K \times 1$  composite vector:

$$Z_{it} = \begin{bmatrix} r_{it} - \bar{r}_{it} \\ X_{it} - \bar{X}_{it} \\ X_{At} - \bar{X}_{At} \end{bmatrix}, \quad (7)$$

where the bar over the variable means the average value across firms and time. We assume this vector evolves according to a VAR(1):

$$Z_{i,t+1} = AZ_{i,t} + \varepsilon_{i,t+1}, \quad (8)$$

where  $\varepsilon_{i,t+1}$  is a vector of conditionally mean-zero, but potentially heteroskedastic, shocks. The companion matrix  $A$  is a  $K \times K$  matrix. Then discount rate shocks are:

$$\begin{aligned} DR_{i,t+1}^{shock} &= E_{t+1} \sum_{j=2}^{\infty} \kappa^{j-1} r_{i,t+j} - E_t \sum_{j=2}^{\infty} \kappa^{j-1} r_{i,t+j} \\ &= e_1' \sum_{j=1}^{\infty} \kappa^j A^j Z_{i,t+1} - e_1' A \sum_{j=1}^{\infty} \kappa^j A^j Z_{i,t} \\ &= e_1' \sum_{j=1}^{\infty} \kappa^j A^j \varepsilon_{i,t+1} = e_1' \kappa A (I_K - \kappa A)^{-1} \varepsilon_{i,t+1}. \end{aligned} \quad (9)$$

Here  $e_1$  is a  $K \times 1$  column vector with 1 as its first element and zeros elsewhere, and  $I_K$  is the  $K \times K$  identity matrix.

We can extract cash flow shocks from the VAR by combining Equation (3) and the

expression for discount rate shocks:

$$\begin{aligned}
CF_{i,t+1}^{shock} &= r_{i,t+1} - E_t[r_{i,t+1}] + DR_{i,t+1}^{shock} \\
&= e'_1 (I_K + \kappa A (I_K - \kappa A)^{-1}) \varepsilon_{i,t+1}.
\end{aligned} \tag{10}$$

Thus, we impose the present-value relation when estimating the joint dynamics of firm cash flows and discount rates.

Note that the companion matrix  $A$  is constant across firms, implying that the firm-level model is a panel VAR(1) as in Vuolteenaho (2002). Identification of the coefficients in  $A$  comes from time-series and cross-sectional variation. Whereas predictive time-series regressions are noisy and often plagued by small-sample problems, such as the Stambaugh (1999) bias, the panel approach alleviates these issues. The cost of the panel assumption is failing to capture some heterogeneity across firms. We minimize this cost by including a broad array of possible determinants of expected returns in  $X_{it}$  and  $X_{At}$  and performing extensive robustness checks. In addition, we do not impose any structure on the error terms across firms or over time since ordinary least squares yields consistent estimates. Instead we adjust standard errors for dependence across firms and time.<sup>7</sup>

We obtain a portfolio-level variance decomposition by aggregating the  $CF_{i,t}^{shock}$  and  $DR_{i,t}^{shock}$  estimates for all firms in a portfolio. Because the firm-level variance decomposition applies to log returns, the portfolio cash flow and discount rate shocks are not simple weighted averages of firms' cash flow and discount rate shocks. Therefore we approximate each firm's gross return using a second-order Taylor expansion around its current expected log return and then aggregate shocks to firms' gross returns using portfolio weights.

The first step in this process is to express gross returns in terms of the components of

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<sup>7</sup>Even if a theoretical model is nonlinear, one can still simulate the model and estimate the VAR that we propose in this paper to test whether the model can explain our empirical findings.

log returns using:

$$\begin{aligned} R_{i,t+1} &\equiv \exp(r_{i,t+1}) \\ &= \exp(E_t r_{i,t+1}) \exp(CF_{i,t+1}^{shock} - DR_{i,t+1}^{shock}), \end{aligned} \quad (11)$$

where  $E_t r_{i,t+1}$  is the predicted log return and  $CF_{i,t}^{shock}$  and  $DR_{i,t}^{shock}$  are estimated shocks from firm-level VAR regressions in which we impose the present-value relation. A second-order expansion around zero for both shocks yields:

$$R_{i,t+1} \approx \exp(E_t r_{i,t+1}) \left\{ 1 + CF_{i,t+1}^{shock} + \frac{1}{2} (CF_{i,t+1}^{shock})^2 - DR_{i,t+1}^{shock} + \frac{1}{2} (DR_{i,t+1}^{shock})^2 + CF_{i,t+1}^{shock} DR_{i,t+1}^{shock} \right\}. \quad (12)$$

Later we show that this approximation works well in practice. Next we define the cash flow and discount rate shocks to firm returns measured in levels as:

$$CF_{i,t+1}^{level\_shock} \equiv \exp(E_t r_{i,t+1}) \left\{ CF_{i,t+1}^{shock} + \frac{1}{2} (CF_{i,t+1}^{shock})^2 \right\}, \quad (13)$$

$$DR_{i,t+1}^{level\_shock} \equiv \exp(E_t r_{i,t+1}) \left\{ DR_{i,t+1}^{shock} - \frac{1}{2} (DR_{i,t+1}^{shock})^2 \right\}, \quad (14)$$

$$CFDR_{i,t+1}^{cross} \equiv \exp(E_t r_{i,t+1}) CF_{i,t+1}^{shock} DR_{i,t+1}^{shock}. \quad (15)$$

For a portfolio with weights  $\omega_{i,t}^p$  on firms, we can approximate the portfolio return measured in levels using:

$$R_{p,t+1} - \sum_{i=1}^n \omega_{i,t}^p \exp(E_t r_{i,t+1}) \approx CF_{p,t+1}^{level\_shock} - DR_{p,t+1}^{level\_shock} + CFDR_{p,t+1}^{cross}, \quad (16)$$

where

$$CF_{p,t+1}^{level\_shock} = \sum_{i=1}^n \omega_{i,t}^p CF_{i,t+1}^{level\_shock}, \quad (17)$$

$$DR_{p,t+1}^{level\_shock} = \sum_{i=1}^n \omega_{i,t}^p DR_{i,t+1}^{level\_shock}, \quad (18)$$

$$CFDR_{p,t+1}^{cross} = \sum_{i=1}^n \omega_{i,t}^p CFDR_{i,t+1}^{cross}. \quad (19)$$

By summing over the individual firms' level cash flow and discount rate shocks, we account for the conditional covariance structure of the shocks when looking at portfolio-level cash flow and discount rate shocks. We decompose the variance of portfolio returns using

$$\begin{aligned} \text{var} \left( \tilde{R}_{p,t+1} \right) &\approx \text{var} \left( CF_{p,t+1}^{level\_shock} \right) + \text{var} \left( DR_{p,t+1}^{level\_shock} \right) \\ &\quad - 2\text{cov} \left( CF_{p,t+1}^{level\_shock}, DR_{p,t+1}^{level\_shock} \right) \\ &\quad + \text{var} \left( CFDR_{p,t+1}^{cross} \right), \end{aligned} \tag{20}$$

where  $\tilde{R}_{p,t+1} \equiv R_{p,t+1} - \sum_{i=1}^n \omega_{i,t}^p \exp(E_t r_{i,t+1})$ . We ignore covariance terms involving  $CFDR_{p,t+1}^{cross}$  as these are very small in practice. When analyzing cash flow and discount rate shocks to long-short portfolios, we obtain the anomaly cash flow (discount rate) shock as the difference in the cash flow (discount rate) shocks between the long and short portfolios.<sup>8</sup>

### 3 Data

We estimate the components in the present-value equation using data from Compustat and Center for Research on Securities Prices (CRSP) from 1962 through 2015. Our analysis requires panel data on firms' returns, book values, market values, earnings, and other accounting information, as well as time-series data on factor returns, risk-free rates, and price indexes. Because some variables require three years of historical data, our VAR estimation focuses on the period from 1964 through 2015.

We obtain all accounting data from Compustat, though we augment our book data with that from Davis, Fama, and French (2000). We obtain data on stock prices, returns, and shares outstanding from CRSP. We obtain one-month and one-year risk-free rate data from one-month and one-year yields of US Treasury Bills, respectively, which are available on

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<sup>8</sup>In Internet Appendix B, we relate the VAR specification to standard asset pricing models, such as Bansal and Yaron (2004). The VAR specification concisely summarizes the dynamics of expected cash flows and returns, even when both consist of multiple components fluctuating at different frequencies. Fundamentally, shocks to firm discount rates arise from shocks to the product of the quantity of firm risk and the aggregate price of risk, as well as shocks to the risk-free rate.

Kenneth French’s website and the Fama Files in CRSP. We obtain inflation data from the Consumer Price Index (CPI) series in CRSP.

We impose sample restrictions to ensure the availability of high-quality accounting and stock price information. We exclude firms with negative book values because we cannot compute the logarithms of their book-to-market ratios as required in the present-value equation. We include only firms with nonmissing market equity data at the end of the most recent calendar year. Firms also must have nonmissing stock return data for at least 225 days in the past year to accurately estimate stock return variance, as discussed below. We exclude firms in the bottom quintile of the size distribution for the New York Stock Exchange to minimize concerns about illiquidity and survivorship bias. Lastly, we exclude firms in the finance and utility industries because accounting and regulatory practices distort these firms’ valuation ratios and cash flows. We impose these restrictions *ex ante* and compute subsequent book-to-market ratios, earnings, and returns as permitted by data availability. We use CRSP delisting returns and assume a delisting return is -90% in the rare cases in which a stock’s delisting return is missing.

When computing a firm’s book-to-market ratio, we adopt the convention of dividing its book equity by its market equity at the end of the June immediately after the calendar year of the book equity. This timing of market equity coincides with the beginning of the stock return period, allowing us to use the clean-surplus equation below. We compute book equity using Compustat data when available, supplementing it with hand-collected data from the Davis, Fama, and French (2000) study. We adopt the Fama and French (1992) procedure for computing book equity. Market equity is equal to shares outstanding times stock price per share. We sum market equity across firms that have more than one share class of stock. We define  $\ln BM$  as the log of book-to-market ratio.

We compute log stock returns in real terms by subtracting the log of inflation (the log change in the CPI) from the log nominal stock return. We compute annual returns from the end of June to the following end of June to ensure that investors have access to December



accounting data prior to the ensuing June-to-June period over which we measure returns.

We measure log clean-surplus return on equity,  $\ln ROE^{CS}$ , from the equation:

$$\ln ROE_{i,t+1}^{CS} \equiv r_{i,t+1} + \kappa b m_{i,t+1} - b m_{i,t}. \quad (21)$$

This measure corresponds to actual return on equity if clean-surplus accounting and the log-linearization both hold, as Ohlson (1995) and Vuolteenaho (2002) assume.<sup>9</sup> It is a timely, June-to-June, earnings measure that exactly satisfies the equation:

$$CF_{i,t+1}^{shock} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa^{j-1} \ln ROE_{i,t+j}^{CS}. \quad (22)$$

Thus, one can use  $\ln ROE^{CS}$  in the VAR to obtain expected cash flows and cash flow shocks at different horizons. In addition, as Equation (21) shows, adding  $\ln ROE^{CS}$  in the VAR is equivalent to including a second lag of the book-to-market ratio. We winsorize the earnings measure at  $\ln(0.01)$  when earnings is less than -99%. We follow the same procedure for log returns and for log firm characteristics that represent percentages with minimum bounds of -100%. Alternative winsorizing procedures have little impact on our results.

We compute several firm characteristics that predict short-term stock returns in historical samples. A firm's market equity (ME) or size is its shares outstanding times share price. Following Fama and French (2015), profitability (Prof) is annual revenues minus costs of goods sold, interest expense, and selling, general, and administrative expenses, all divided by book equity from the same fiscal year.<sup>10</sup> Following Cooper, Gulen, and Schill (2008) and Fama and French (2015), investment (Inv) is the annual percentage growth in total assets. Annual data presents a challenge for measuring the momentum anomaly. In Jegadeesh and Titman (1993), the maximum momentum profits accrue when the formation and holding periods sum to 15 to 18 months. Therefore, we construct a six-month momentum variable based on the percentage rank of each firm's January to June return. The subsequent holding

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<sup>9</sup>Violations of clean-surplus accounting can arise from share issuance or merger events.

<sup>10</sup>Novy-Marx (2013) defines profitability with a denominator of total assets, not book equity.

period implicit in the VAR is one year, from July through June. We transform each measure by adding one and taking its log, resulting in the following variables:  $\ln ME$ ,  $\ln Prof$ ,  $\ln Inv$ , and  $\ln Mom6$ . We also subtract the log of gross domestic product from  $\ln ME$  to ensure stationarity. We use another stationary measure of firm size ( $SizeWt$ ), equal to firm market capitalization divided by the total market capitalization of all firms in the sample, when applying value weights to firms' returns in portfolio formation.

We compute stocks' annual return variances based on daily excess log returns, which are daily log stock returns minus the daily log return from the one-month risk-free rate at the beginning of the month. A stock's realized variance is the annualized average value of its squared daily excess log returns during the past year. In this calculation, we do not subtract each stock's mean squared excess return to minimize estimation error. We transform realized variance by adding one and taking its log, resulting in the variable  $\ln RV$ .

Table 1 presents summary statistics for key variables. For ease of interpretation, we show statistics for nominal annual stock returns ( $AnnRet$ ), nominal risk-free rates ( $Rf$ ), and inflation ( $Inflat$ ) before we apply the log transformation. Similarly, we summarize stock return volatility ( $Volat$ ) instead of log variance. We multiply all statistics by 100 to convert them to percentages, except  $\ln BM$  and  $\ln ME$ , which retain their original scale.

Panel A displays the number of observations, means, standard deviations, and percentiles for each variable. The median firm has a log book-to-market ratio of  $-0.66$ , which implies a market-to-book ratio of  $e^{0.66} = 1.94$ . Valuation ratios range widely, as shown by the 10th and 90th percentiles of market-to-book ratios of 0.75 and 5.93. The variation in stock returns is substantial, ranging from  $-40\%$  to  $66\%$  for the 10th and 90th percentiles. Panel B shows correlations among the accounting characteristics in the VAR, which are all modest.

## 4 VAR Estimation

We estimate the firm-specific and aggregate predictors of firms' (log) returns and cash flows using a panel VAR system. Natural predictors of returns include characteristics that are

proxies for firms' risk exposures or stock mispricing. As predictors of earnings, we use accounting characteristics and market prices that forecast firm cash flows in theory and practice.

## 4.1 Specification

Our primary VAR specification includes eight firm-specific characteristics: firm returns and clean-surplus earnings ( $\ln Ret$  and  $\ln ROE^{CS}$ ), five anomaly characteristics ( $\ln BM$ ,  $\ln Prof$ ,  $\ln Inv$ ,  $\ln ME$ , and  $\ln Mom6$ ), and log realized variance ( $\ln RV$ ). We include  $\ln RV$  to capture omitted factor exposures and differences between expected log returns and the log of expected returns. We standardize each independent variable by its full-sample standard deviation to facilitate interpretation of the regression coefficients. The only exceptions are  $\ln BM$ ,  $\ln Ret$ , and  $\ln ROE^{CS}$ , which we do not standardize to enable imposing the present-value relation in the VAR estimation. All log return and log earnings forecasting regressions include the log real risk-free rate ( $\ln Rf$ ) to capture common variation in firm valuations resulting from changes in market-wide discount rates. Finally, we add interactions of the anomaly characteristics ( $\ln BM$ ,  $\ln Prof$ ,  $\ln ME$ ,  $\ln Inv$ , and  $\ln Mom6$ ) with  $\ln BM$ . For each characteristic, the interaction is the product of  $\ln BM$  and a variable that equals 1 if a stock is in the top quintile of the characteristic, -1 if a stock is in the bottom quintile of the characteristic, and 0 otherwise. This specification allows the coefficient on  $\ln BM$  to be different for stocks in each leg of the long-short anomaly portfolios and for stocks not in these extreme portfolios.

We estimate a first-order autoregressive system with one lag of each characteristic. This VAR allows us to estimate the long-run dynamics of log returns and log earnings based on the short-run properties of a broad cross section of firms. We do not need to impose restrictions on which firms survive for multiple years, thereby mitigating statistical noise and survivorship bias. As a robustness check, we estimate a second-order VAR and find similar results as the second lags of characteristics add little explanatory power.

The VAR system also includes forecasting regressions for firm and aggregate variables. We regress  $\ln Ret$ ,  $\ln ROE^{CS}$ , and  $\ln BM$  on all characteristics. For each other characteristic, the only predictors are the lagged characteristic and lagged log book-to-market ratio. For example, the only predictors of log investment are lagged log investment and lagged log book-to-market ratio. This restriction improves estimation efficiency without significantly reducing explanatory power. We model the real risk-free rate as a first-order autoregressive process.

Our primary VAR specification omits aggregate variables other than the risk-free rate, raising the concern that we are missing a common component in firms' expected cash flows and discount rates. In Section 7, we consider VAR specifications that include the market-wide valuation ratio and its interactions with firm characteristics. We show these additional variables do not materially increase the explanatory power of our regressions and result in extremely high standard errors in return variance decompositions. Section 7 also discusses the implications of data mining characteristics and industry fixed effects in characteristics. We conduct all tests using standard equal-weighted regressions, but our findings are robust to applying value weights to each observation. Overall, our findings are robust to several alternative specifications.

## 4.2 Baseline Panel VAR Estimation

The first two columns of Table 2 report the coefficients in the forecasting regressions for firms' log returns and earnings. The third column in Table 2 shows the implied coefficients on firms' log book-to-market ratios based on the clean-surplus relation between log returns, log earnings, and log valuations (see Equation (21)). We use OLS to estimate each row in the  $A$  matrix of the VAR. Standard errors clustered by year and firm, following Petersen (2009), appear in parentheses below the coefficients.

The findings in the log return regressions are consistent with those of the large literature on short-horizon forecasts of returns. Firms' log book-to-market ratios and profitability

are positive predictors of their log returns at the annual frequency, whereas log investment is a negative predictor of log returns. Log firm size and realized variance weakly predict returns with the expected negative signs, while momentum has a positive sign, though these coefficients are not statistically significant in this multivariate panel regression. The largest standardized coefficients are those for firm-specific log book-to-market (0.042), profitability (0.043), and investment ( $-0.051$ ).<sup>11</sup> These coefficients represent the change in expected annual return from a one standard deviation change in each characteristic holding other predictors constant.

The second column of Table 2 shows the regressions predicting log annual earnings. One of the strongest predictors of log earnings is log book-to-market, which has a coefficient of  $-0.109$ . Other predictors of log earnings include the logs of returns, profitability, past earnings, and several of the book-to-market interaction terms.

The third column in Table 2 shows how lagged characteristics predict log book-to-market ratios. Log book-to-market ratios are quite persistent as shown by the 0.875 coefficient on lagged  $\ln BM$ . This high persistence coupled with the strong predictive power of  $\ln BM$  for earnings and returns suggests that  $\ln BM$  is an important determinant of cash flow and discount rate news. Interestingly, log investment is a significant positive predictor of log book-to-market, meaning that market-to-book ratios tend to decrease following high investment. These relations play a role in the long-run dynamics of expected log earnings and log returns of firms with high investment. Analogous reasoning applies to the positive coefficient on lagged log returns, which is statistically significant at the 10% level.

Table 3 shows regressions of firm characteristics on lagged characteristics and lagged book-to-market ratio. The most persistent characteristic is log firm size, which has a persistence coefficient of 0.978. We can, however, reject the hypothesis that this coefficient is 1.000. The persistence coefficients on the logs of profitability and realized variance are 0.734

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<sup>11</sup>Since log book-to-market ratios are not standardized in the VAR, the actual regression coefficient reported in Table 2 is 0.051, which is the standardized coefficient of 0.042 divided by the standard deviation of log book-to-market ratios of 0.83.

and 0.688, respectively, whereas the persistence coefficients on the logs of investment and momentum are just 0.157 and 0.048, respectively. All else equal, characteristics with high (low) persistence are more important determinants of long-run cash flows and discount rates. Lagged log book-to-market is a significant predictor of the logs of profitability, investment, momentum, and realized variance, but the incremental explanatory power from lagged valuations is modest in all cases except the investment regression. Table 3 also shows that the lagged real risk-free rate ( $\ln Rf$ ) is reasonably persistent with a coefficient of 0.603. This estimate has little impact on expected long-run returns and cash flows because the risk-free rate is not a significant predictor of returns or cash flows, as shown in Table 2.

## 5 Firm-level Analysis

We now examine the decomposition of firms' log book-to-market ratios and returns implied by the regression results. We first analyze the correlations and covariances between total log book-to-market ( $\ln BM$ ) and its cash flow (CF) and discount rate (DR) components. Table 4 shows that DR and CF variation respectively account for 22.5% and 53.3% of return variation, confirming the finding in Vuolteenaho (2002) that firm-level returns are driven mainly by cash flow shocks. Interestingly, covariation between DR and CF tends to amplify return variance, contributing a highly significant amount (24.3%) of variance. The last column shows that the correlation between the CF and DR components is significantly negative ( $-0.351$ ). In economic terms, this correlation means that low expected cash flows are associated with high discount rates. The negative correlation in cash flow and discount rate shocks could arise for behavioral or rational reasons. Investor overreaction to positive firm-specific cash flow shocks could lower firms' effective discount rates. Alternatively firms with negative cash flow shocks could become more exposed to systematic risks, increasing their discount rates.

## 6 Portfolio-level Analysis

Now we analyze the implied discount rate (DR) and cash flow (CF) variation in returns to important portfolios, including the market portfolio and anomaly portfolios formed by cross-sectional sorts on value, size, profitability, investment, and momentum. We compute weighted averages of firm-level DR and CF estimates to obtain portfolio-level DR and CF estimates. We apply the approximation and aggregation procedure described in Section 2. When aggregating firm-level shocks to the portfolio level, only correlated shocks to firms remain. Thus, if cash flow shocks are largely uncorrelated but discount rate shocks are highly correlated, the portfolio return variance decomposition can be very different from the firm return variance decomposition.

### 6.1 The Market Portfolio

We define the market portfolio as the value-weighted average of individual firms. We compare the estimates from our aggregation approach to those from a standard aggregate-level VAR in the spirit of Campbell (1991). In the aggregate VAR, we use only the logs of (market-level) book-to-market ratio ( $AgglnBM$ ) and the real risk-free ( $lnRf$ ) as predictors of the logs of market-level earnings and returns. Accordingly, this specification entails just three regressions in which market-level earnings, returns, and risk-free rates are the dependent variables and lagged book-to-market and risk-free rates are the independent variables.

We next validate our panel VAR approach and compare it to the market-level VAR by assessing model predictions of long-run outcomes. Although long-run expected cash flows and returns form the basis for the return decomposition, VAR estimation only maximizes short-run forecasting power and could produce poor long-run forecasts. We define long-run

expected cash flows and discount rates as:

$$\begin{aligned} CF_{i,t}^{LR} &= E_t \sum_{j=1}^{\infty} \kappa^{j-1} \ln ROE_{i,t+j}^{CS}, \\ DR_{i,t}^{LR} &= E_t \sum_{j=1}^{\infty} \kappa^{j-1} r_{i,t+j}. \end{aligned} \tag{23}$$

Given the definition of clean-surplus earnings in Equation (21), we have that:

$$bm_{i,t} = DR_{i,t}^{LR} - CF_{i,t}^{LR}. \tag{24}$$

By the present value restriction, the difference between these long-run discount rate and cash flow components must equal current log book-to-market. These valuation components should, if the VAR accurately describes long-run dynamics, forecast realized long-run market earnings and returns. Since we cannot compute infinite-horizon earnings and returns, we forecast 10-year log market earnings (returns) using the long-run cash flow (discount rate) component from the VAR.

Figure 1 shows predicted versus realized market earnings and returns over the next 10 years. We construct the series of 10-year realized earnings (returns) based on firms' current market weights and their future 10-year earnings (returns). Thus, we forecast 10-year buy-and-hold returns to the market portfolio, not the returns to an annually rebalanced trading strategy. We do not rebalance the portfolio because the underlying discount rate estimates from the panel VAR are specific to firms. This distinction is important insofar as firm entry, exit, issuance, and repurchases occur.

The top plot in Figure 1 shows predicted long-run market earnings from our panel VAR (dashed red line) and the market-level VAR (dotted black line). Both predictions track realized 10-year market earnings well, with a somewhat higher  $R^2$  for the panel VAR (73%) than for the market VAR (58%). The bottom plot in Figure 1 shows that predictions of long-run returns from the two VARs are similar, except that the panel VAR predicts lower returns around the year 2000. Both sets of predictions are significantly correlated with realized 10-



year returns. The panel VAR  $R^2$  is 41%, whereas the market-level VAR  $R^2$  is 19%. These plots suggest that both VAR methods yield meaningful decompositions of valuations into CF and DR components. Even though the panel VAR does not directly analyze the market portfolio, aggregating the panel VAR’s firm-level predictions results in forecasts of market cash flows and returns that slightly outperform forecasts based on the traditional approach.

Next we compare the two VARs’ implied decompositions of market return variance. We compute market cash flow and discount rate shocks from both VARs, as in Equations (9) and (10) in Section 2, and analyze the covariance matrix of these shocks. When calculating the aggregated panel VAR news from time  $t$  to time  $t + 1$ , the updated expectation is based on the firms in the market portfolio at time  $t$ .

Table 4 presents variance decompositions of market returns based on the panel VAR and the time-series VAR. The first four columns decompose the variance of predicted market returns from our approximation into four nearly exhaustive components: the variances of DR, CF, and the cross term (CF\*DR), and the covariance between CF and DR. We do not report the covariances between the cross term and the CF and DR terms because they are negligible. We normalize all quantities by the variance of unexpected returns, so the variance components sum to one. The fifth column in Table 4 reports the correlation between market DR and CF news. The last column shows that the correlation between our approximation of market returns and actual market returns is 0.985, indicating that our approximation is accurate. Standard errors based on the delta method appear in parentheses.

Table 4 shows that the panel and market-level VARs predict similar amounts of discount rate variation (18% and 28%, respectively), but the estimate from the panel VAR is more precise judging by its standard error. Both estimates of DR variation seem lower than those reported in prior studies for two reasons. First, Cochrane (2011) decomposes log return variance ( $var(r)$ ) into  $cov(CF, r)$  and  $cov(-DR, r)$ , whereas our decomposition explicitly accounts for the covariance term following Campbell (1991). Using Cochrane’s (2011) alternative decomposition would increase discount rate news to 33% and 52% for the panel and

market VARs, respectively. Second, the samples are different. Restricting the sample of the market VAR to 1964 to 1990 increases in the magnitude of DR variation.

Our estimates from the panel VAR imply that market cash flow news accounts for 55% of market return variance, whereas the market-level VAR implies that CF news explains just 25% of return variance. The two VARs also differ in the implied correlations between CF and DR news. The panel VAR indicates that the correlation is just  $-0.492$ , whereas the market-level VAR implies a correlation of  $-0.892$ .

One reason for the discrepancy is that the panel VAR employs far more predictive variables than the market-level VAR, leading to a more accurate description of expected cash flows and discount rates. A second reason is that the market VAR suffers from two biases induced by the time-series properties of aggregate book-to-market ratios (AgglBM): 1) this highly persistent regressor causes a substantial Stambaugh (1999) bias in the relatively short sample; and 2) an apparent structural break in AgglBM occurs around 1990, as noted by Lettau and van Nieuwerburgh (2008) in the context of the market price-dividend ratio, implying that AgglBM is effectively non-stationary variable in this sample. Based on these considerations, we exclude AgglBM from our baseline panel VAR specification, though we consider its impact on our conclusions in alternative specifications discussed in Section 7.

## 6.2 Anomaly Portfolios

We now analyze the returns of long-short anomaly portfolios to bring new facts to the debate on the source of anomalies. We estimate the cash flow and discount rate components of historical anomaly returns and analyze the covariance matrix of these shocks. We then evaluate whether theories of anomalies make reasonable predictions about the cash flow and discount rate components of anomaly returns. Further, we consider the MVE combination of anomaly portfolios and the market portfolio to decompose priced risks.

Anomaly portfolios represent trading strategies. The underlying firms in these portfolios change every year based on firms' characteristic rankings in June. However, for any given

year, the portfolio return is driven by the cash flow and discount rate shocks of the underlying firms in the portfolio in that year. The firm-level VAR allows us to relate anomaly returns to underlying firm fundamentals. We aggregate the firm-level estimates using value weights within each quintile and then analyze portfolios with long positions in quintile 5 and short positions in quintile 1 according to firms' characteristic rankings. The aggregation procedure is otherwise analogous to that used for the market portfolio.

We validate anomaly portfolio components just as we did for the market. The plots in Figure 2 show that the estimated long-run cash flow and discount rate components of the long-short value portfolio indeed forecast the respective 10-year earnings and returns of this portfolio. In the top plot in Figure 2, the predictor is the difference between the long-run CF and DR components of value and growth firms, and realized cash flows represent the difference in 10-year earnings of value and growth firms. The plot shows that predicted earnings are correlated with future 10-year earnings, primarily in the second half of the sample. The overall  $R^2$  is modest at 24%. The bottom plot in Figure 2 depicts the relationship between the DR component of the long-short value portfolio and future 10-year returns. This relationship is strong in both halves of the sample, and the overall  $R^2$  is high at 48%.

Figure 3 presents analogous  $R^2$  statistics for the long-run cash flow and discount rate components of all five long-short anomaly portfolios and the market portfolio. The long-run DR component of the size anomaly portfolio forecasts its 10-year returns quite well ( $R^2 = 61\%$ ), whereas the long-run DR component of the momentum portfolio has modest forecasting power for 10-year momentum returns ( $R^2 = 16\%$ ). The  $R^2$  values in Figure 3 range from 16% to 73%, implying correlations between the long-run CF and DR components and their realized counterparts that range between 0.40 and 0.85. We conclude from this analysis that the aggregated cash flow and discount rate components are robust predictors of anomaly portfolios' long-term earnings and returns.

### 6.2.1 Anomaly and MVE Portfolio Variance Decompositions

Panel A of Table 5 presents variance decompositions of anomaly returns for the five anomalies and is analogous to Table 4 for the market. Table 5 reveals remarkably consistent patterns across the five anomaly portfolios. Cash flow variation accounts for 44% to 56% of variation in anomaly returns, whereas discount rate variation accounts for just 16% to 23%. The covariance between CF and DR is consistently negative, and this covariance term accounts for 32% to 37% of anomaly return variance. The standard errors on the CF and DR variances range from 10% to 17%, indicating the high precision of these findings. The cross term (CF\*DR) accounts for only 2% to 4% of anomaly return variance. The last column shows that the correlation between approximate anomaly returns and actual anomaly returns ranges from 0.88 to 0.96, indicating the approximation is accurate.

The importance of cash flows and the negative correlation between CF and DR are the most prominent effects. Theories of anomalies that rely heavily on independent variation in DR shocks, such as De Long et al. (1990), are inconsistent with the evidence in Table 5. In contrast, theories in which CF shocks are tightly linked with DR shocks have the potential to explain this evidence. Rational theories in which firm risk increases after negative cash flow realizations predict negative correlations between CF and DR shocks. Behavioral theories in which investors overreact to cash flow news are also consistent with this evidence.

The similarity of the empirical decompositions across anomalies is not mechanical. Aggregation into long-short portfolios diversifies away idiosyncratic cash flow and discount rate shocks, focusing the analysis on common cash flow and discount rate variation in anomaly portfolios. The relative importance of CF and DR for each anomaly depends on the correlation of CF and DR shocks across the assets in anomaly portfolios, which in turn depends on the commonality in shocks to assets' characteristics. Furthermore, we include interaction terms in the VAR to allow the coefficient on lagged book-to-market—the most important determinant of CF and DR—to vary by anomaly.

Panel B of Table 5 decomposes return variance of in-sample MVE portfolios. We com-

pute MVE portfolio weights using the standard formula for maximizing Sharpe ratio based on assets' average returns and covariance matrix. The first row in Table 5 reports the decomposition for the ex post MVE portfolio comprising only the five long-short anomaly portfolios. An arbitrageur would hold this portfolio if anomalies arise from mispricing. For this anomaly MVE portfolio, discount rate news accounts for 14% of return variation, cash flow news accounts for 44%, and negative covariation between CF and DR news accounts for 37%. The CF and DR news correlation is  $-0.75$ . Thus, aggregating across anomalies does not materially affect the variance decomposition. Cash flow shocks are still the most important contributor to variance, and the correlation between cash flow and discount rate shocks becomes even more negative. The second row shows the in-sample MVE portfolio that includes the market portfolio. In theory, this portfolio's return covaries most negatively with the marginal agent's marginal utility. Cash flow variation for this MVE portfolio is even stronger at 58%. Discount rate variation accounts for 13% of return variance, and the correlation between CF and DR news is still significantly negative at  $-0.58$ .

### **6.2.2 Correlation Between Market and Anomaly Portfolio News**

Panel A of Table 6 displays correlations between the components of market returns and those of anomaly returns. The four columns indicate the correlations between market cash flow and discount rate shocks and anomaly cash flow and discount rate shocks. Standard errors based on the delta method appear in parentheses.

Strikingly, none of the five anomaly cash flow shocks exhibits a significant correlation with market cash flows, as shown in column one. The five correlations between anomaly and market cash flows range between  $-0.12$  and  $0.08$  and are statistically indistinguishable from zero. These findings cast doubt on theories of anomalies that rely on differences in how sensitive firms' earnings are to aggregate cash flows. The evidence is ostensibly inconsistent with a broad category of risk-based explanations of anomalies.

Cohen, Polk, and Vuolteenaho (2009) argue that market cash flow betas are increasing

in firms' book-to-market ratios. Their results differ from ours for three reasons. First, our sample is longer and includes the financial crisis. Second, we exclude the bottom size quintile of firms. Third, our cash flow correlations are based on infinite-horizon cash flows as defined by the return decomposition that imposes the present value constraint. The main cash flow betas in Cohen, Polk, and Vuolteenaho (2009) are, in contrast, based on 5-years of ROE data. In Panel A of Table 2 in their paper, they find that cash flow betas of the long-short market-to-book portfolio (the 10-1 column) decrease in absolute value as horizons increase beyond five years. At the 15-year horizon, this cash flow beta falls by 80% and becomes statistically insignificant. Extrapolating from this table, our results in fact appear consistent with those of Cohen, Polk, and Vuolteenaho (2009), though our interpretation is somewhat different.

The fourth column in Table 6 reveals that discount rate shocks to four of the five anomalies have insignificant correlations with discount rate shocks to the market. However, DR shocks to the profitability anomaly are significantly negatively correlated ( $-0.45$ ) with DR shocks to the market. One interpretation is that increases in the market-wide cost of capital are associated with a flight to quality in which investors become relatively eager (reluctant) to provide funding to firms with high (low) profits.

The second and third columns in Table 6 reveal a few notable cross-correlations between market and individual anomaly CF and DR shocks. Three of these correlations exceed 0.3 in absolute value and are economically material and statistically significant at the 5% level. The positive correlation of 0.36 between the CF shock to the market and the (negative) DR shock to the size portfolio suggests that small firms have lower costs of capital during economic booms. The correlations between CF shocks to the value and investment portfolios and DR shocks to the market indicate that firms with high valuations and high investment have higher expected cash flows when market-wide discount rates fall. Campbell, Polk, and Vuolteenaho (2010) also find that the long-short value portfolio has cash flow shocks that are positively correlated with market discount rate shocks. They find a small but significantly positive correlation between this portfolio's cash flow shocks and market cash flow shocks,

while we find that this correlation is slightly negative but insignificant. Our sample period includes the financial crisis, while theirs does not. Their standard errors do not account for estimation uncertainty in the VAR coefficient estimates, while ours do. Aside from the notable correlations described above, most of the correlations between market and anomaly return components are low, which is consistent with theories in which idiosyncratic cash flow shocks affect firms' expected returns—e.g., Babenko, Boguth, and Tserlukevich (2016).

Panel B of Table 6 shows correlations between shocks to the market portfolio and shocks to the anomaly MVE portfolio. The correlation of anomaly CF shocks with market CF shocks is close to zero ( $-0.01$ ), suggesting that market cash flow betas cannot explain anomalies' expected returns. The correlation between anomaly MVE and market DR shocks is actually negative at  $-0.27$ , casting doubt on the idea that arbitrageurs exploiting anomalies are exposed to the same shocks to risk preferences as investors holding the market. Instead, the evidence suggests distinct forces drive market and anomaly return components.

Generalizing from the last column in Table 6, the weak correlation between most anomalies' DR shocks and market DR shocks is inconsistent with theories of common DR shocks. In theories such as Campbell and Cochrane (1999), commonality in DR shocks occurs because risk aversion varies over time. Similarly, theories in which anomalies and the market are driven by common shocks to investor sentiment, such as Baker and Wurgler (2006), are at odds with the low empirical correlation in anomaly and market DR shocks. In Section 7, we will show that the key patterns in Table 6 are robust to measuring market CF and DR shocks using the traditional time-series VAR rather than aggregating the panel VAR shocks.

To explore these relations further, Figure 4 plots anomaly MVE and market CF shocks in Panel A and the corresponding DR shocks in Panel B. In the financial crisis of 2008-2009, the market and anomaly MVE portfolios both experience negative CF shocks; both portfolios also experience positive DR shocks, though this effect is more pronounced for the anomaly portfolio. In contrast, during the dot-com boom of the late 1990s and the ensuing crash, the market and anomaly MVE DR shocks diverge from each other. In the boom,

market DR shocks were negative, while anomaly MVE DR shocks were positive, with the opposite pattern holding for the crash. This pattern reflects the success of high investment, low profitability, and low book-to-market firms during the dot-com boom, and their poor performance during the crash.

### 6.3 Short-run vs. Long-run Cash Flow Shocks

We now decompose cash flow shocks into short-run (one-year) and long-run cash flow shocks. For each firm  $i$ , we define the short-run cash flow shock as:

$$CF_{i,t+1}^{short-run} = \ln ROE_{i,t+1}^{CS} - E_t [\ln ROE_{i,t+1}^{CS}],$$

where the expected cash flow comes from the VAR. The long-run cash flow shocks is then:

$$CF_{i,t+1}^{long-run} = CF_{i,t+1}^{shock} - CF_{i,t+1}^{short-run}.$$

We aggregate firm-level CF shocks into portfolio CF shocks for long-short anomalies and the market using the same procedure applied to the total cash flow shock,  $CF_{i,t+1}^{shock}$ .

Table 7 shows that the negative correlation between cash flow and discount rate shocks is due to the long-run cash flow shock,  $CF_{i,t+1}^{long-run}$ . The correlations between the long-run cash flow shocks and discount rate shocks of each anomaly are strongly negative. With the exception of the profitability anomaly, all of these correlations are less than  $-0.5$  and are statistically significant at the 1% level. In contrast, the correlations between the short-run cash flow shocks and discount rate shocks are statistically insignificant and economically small for all anomalies. These facts support models in which correlated shocks to long-run firm earnings drive discount rate shocks, such as the model of investor overconfidence of Daniel, Hirshleifer, and Subrahmanyam (2001). For the market portfolio, none of the correlations are statistically significant. However, the signs and magnitudes of the market portfolio's short- and long-run cash flow correlations are similar to those of the anomaly portfolios: discount rates exhibit no correlation with short-run cash flow shocks and a large



negative correlation with long-run cash flow shocks.

## 6.4 Correlations with Aggregate Shocks

In Table 8, we report contemporaneous correlations of CF and DR shocks to the market and anomaly portfolios with notable aggregate shocks. We estimate each aggregate shock as the residuals from an AR(1) model of the relevant series. One group of aggregate shocks reflects macroeconomic cash flow shocks: real per-capita consumption and GDP growth, 3-year forward-looking consumption growth, and the labor share. The other group represents shocks to aggregate risk aversion or discount rates: one-year change in the default spread (Baa - Aaa corporate bonds); one-year change in three-month T-bill rate; change in one-year sentiment (obtained from Jeff Wurgler's website, 1965 to 2010); and the broker-dealer leverage factor of Adrien, Etula, and Muir (2014).

Consistent with intuition, market CF shocks are positively correlated with macroeconomic cash flows, namely consumption and GDP growth. Positive shocks to the labor share and thus negative shocks to the capital share are negatively correlated with market CF shocks. Market CF shocks are negatively correlated with shocks to the default spread, which is a measure of risk aversion or discount rates. Market CF shocks are also significantly positively related to broker-dealer leverage shocks, implying that broker-dealers increase leverage in good times. Market DR shocks are marginally significantly correlated with consumption growth, but none of the other aggregate shocks. Based on point estimates, market discount rates decrease when consumption growth is high, which makes economic sense.

We observe two main patterns in anomaly CF and DR shocks. First, the broker-dealer leverage shock exhibits a consistent relation with all anomalies. When broker-dealers increase leverage, presumably in good times, anomaly cash flows are higher and discount rates are lower. Second, anomaly CF and DR shocks are generally uncorrelated with other macroeconomic shocks. A minor exception is the change in labor share, which is positively correlated with discount rate shocks to the book-to-market and size portfolios.

Overall, the anomaly CF and DR shocks exhibit inconsistent correlations with aggregate shocks, except the broker-dealer shock. However, if we interpret the broker-dealer leverage shock as a shock to arbitrageurs' wealth, these correlations do not by themselves suggest a fundamental explanation for anomaly CF and DR comovement.

In our view, the evidence points towards a theory in which investors overextrapolate long-run cash flow news and firms' cash flow exposures are correlated with anomaly characteristics. For example, a technology shock could increase growth firms' cash flows and decrease value firms' cash flows. Investor overreaction to this technology shock would reduce growth firms' discount rates and increase value firms' discount rates. Alternatively, a rational theory in which a technology shock decreases growth firms' risks and increases value firms' risks could be consistent with the evidence. Such cash flow shocks could not be market-level shocks or industry shocks, as they exhibit low correlations with market CF and DR shocks and industry exposures do not appear to be priced. The Kogan and Papanikolaou (2013) model is consistent with these facts.

## **6.5 Overfitting and Misspecifying Expected Returns**

Some characteristics could appear to predict returns only because researchers mining the data for return predictability eventually will find characteristics that appear to predict returns in historical samples. Importantly, using data-mined characteristics in our VAR framework would bias estimates of return predictability upward. As a result, data mining increases the volatility of discount rate shocks. In addition, data mining increases the correlation between implied cash flow shocks and discount rate shocks because CF shocks must offset the impact of DR shocks in total returns, which we observe directly. Our main findings indicate that cash flow shocks are the dominant component of anomaly returns and that the correlation between discount rate and cash flow shocks is negative. Without data mining of VAR characteristics, these two main conclusions would be even more pronounced.

The impact of a specification error in our model of expected returns is analogous to that

of data mining. For example, suppose that we incorrectly identify discount rate shocks, observing  $DR_{i,t+1}^{shock} = trueDR_{i,t+1}^{shock} + error$ . The present-value equation implies that our cash flow shocks inherit the same specification error because  $CF_{i,t+1}^{shock} = r_{i,t+1} - E_t[r_{i,t+1}] + trueDR_{i,t+1}^{shock} + error$ . Thus, any specification error tends to increase the correlation between CF and DR shocks, making our empirical finding of a strong negative correlation even harder to explain.

## 7 Alternative Specifications

Here we consider five alternative specifications of the firm-level VAR in which we include different predictors of cash flows and returns. The first alternative specification (*Spec1*) uses the same predictors as our main specification (*Spec0*), except that it excludes the interaction terms between valuation ratios and anomaly characteristics. Because this specification is the most parsimonious, we use *Spec1* as a baseline for all other alternative specifications.

The second alternative specification (*Spec2*) adds only the market-wide book-to-market ratio, as measured by the value-weighted average of sample firms' log book-to-market ratios, to our main specification (*Spec1*). The third alternative specification (*Spec3*) augments *Spec1* by including interaction terms between market-wide valuations and firm characteristics, including the five anomaly characteristics and realized variance. Market valuations could capture common variation in firms' cash flows and discount rates and interactions with firm characteristics could capture firms' differential exposures to market-wide variation.

The estimation of the key return and cash flow forecasting regressions in the VAR indicates that these additional regressors only modestly contribute to explanatory power. The adjusted  $R^2$  in the return regression increases from 4.6% in *Spec1* to 5.5% in *Spec2*, and the coefficient on the added market-wide valuation variable is only marginally statistically significant ( $p\text{-value} = 0.053$ ). In the earnings regression, the coefficient on market-wide valuation is statistically significant at the 1% level, but the adjusted  $R^2$  barely increases from 24.3% in *Spec1* to 24.9% in *Spec2*. The findings for the third alternative specification, *Spec3*, sug-

gest that the six interaction terms do not contribute incremental explanatory power beyond *Spec2*. Specifically, the adjusted  $R^2$  for the return and earnings regressions are equal to or less than those for *Spec2* and the vast majority of the interaction coefficients are statistically insignificant. Overall, these regressions provide scant evidence that the most parsimonious specification, *Spec1*, is misspecified.

We now evaluate the implications of the alternative specifications for return variance decompositions. Table 9 shows the components of market return variance implied by *Spec2* and *Spec3*. The difference between Table 9 and Table 4, which shows the results for *Spec0*, is striking. Whereas discount rate variation accounts for just 23% of return variance in *Spec1*, it accounts for 107% and 150% of variation in *Spec2* and *Spec3*, respectively. The main reason is that high market-wide book-to-market ratios forecast higher returns and such valuations ratios are highly persistent, implying that their long-run impact is large. However, this predictive relationship is very weak statistically, so the standard errors on the variance decompositions are enormous in *Spec2* and *Spec3*. In fact, in both cases, we cannot reject the hypothesis that DR variation accounts for 0% of variation in returns. Thus, the striking differences in point estimates across the specifications do not necessarily imply strikingly different conclusions.

Table 10 shows the components of anomaly return variance implied by alternative VAR specifications. Importantly, cash flow variation accounts for the highest share of anomaly return variance in all specifications. The finding that discount rates are negatively correlated with expected cash flows also generalizes from *Spec1* to *Spec2*. In *Spec3*, which allows for interaction terms between market-wide valuations and firm-level characteristics, the standard errors are too large to draw reliable inferences about the CF-DR correlation.

To assess which VAR specification provides the most meaningful decomposition of market and anomaly returns, we analyze the long-term forecasting power implied by each specification. Figure 5 shows the long-run forecasts of market earnings and returns from *Spec2*, just as Figure 1 shows these forecasts for our main panel specification (*Spec0*). Although adding

market-wide valuations slightly improves the forecasting power in the one-year earnings regression, long-run predictions based on the *Spec2* model are vastly inferior to those based on the more parsimonious *Spec1* model. The adjusted  $R^2$  values of 73% for *Spec0*, compared to just 0.3% for *Spec2*, confirm the visual impression from the figures. The two specifications exhibit little difference in their ability to predict 10-year market returns ( $R^2 = 41\%$  for *Spec0* vs.  $R^2 = 33\%$  for *Spec2*).

For each of the five alternative specifications, Figure 6 shows the 10-year forecasting power ( $R^2$ ) for market and anomaly earnings and returns. All specifications exhibit similar abilities to predict long-term anomaly returns and earnings, which is not surprising in light of the similar anomaly return decompositions arising from these specifications. The most notable difference arises in the forecasting power for market earnings. Both specifications that include market-wide valuations give rise to very poor forecasts of 10-year market earnings. Apparent structural breaks in market-wide valuations, such as those proposed by Lettau and van Nieuwerburgh (2008), could help explain the poor long-term forecasting power of these two VAR specifications. We conclude that our baseline panel VAR (*Spec0*) not only gives rise to the most precise estimates of market and anomaly return components, but it also exhibits the most desirable long-term forecasting properties.

Lastly, we consider two specifications designed to capture industry and firm fixed effects.<sup>12</sup> However, as noted by Nickell (1981), including fixed effects in a dynamic panel regression, such as that in our main specification, leads to severely biased coefficients in small  $T$  large  $N$  settings such as ours. The biases are similar to the familiar Stambaugh bias that arises in time-series VARs with return forecasting regressions. To avoid this statistical issue and to mimic investor learning, we include rolling means of firm or industry clean-surplus earnings and book-to-market ratios in the VARs. Because we use data until time  $t$  to compute the mean at time  $t$ , including these quasi-fixed effects does not induce a mechanical small-sample correlation between the shocks and the explanatory variables, which would arise with actual

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<sup>12</sup>In unreported tests, we explore specifications that include additional market-level and anomaly-level variables, such as aggregated anomaly characteristics and spreads in valuations across anomaly portfolios.

fixed effects. We apply shrinkage to these rolling means to increase the precision of our estimates. Tables 9 and 10 show the resulting variance decompositions. The case with industry rolling means is *Spec4*, and the case with firm-specific rolling means is *Spec5*. Overall, the inclusion of these variables does not materially alter the anomaly decomposition results relative to our baseline specification (*Spec0*).

## 8 Conclusion

Despite decades of research on predicting short-term stock returns, there is no widely accepted explanation for observed cross-sectional patterns in stock returns. We provide new evidence on the sources of anomaly portfolio returns by aggregating firm-level cash flow and discount rate estimates from a panel VAR system. This aggregation approach enables us to study the components of portfolio returns, while avoiding the biases inherent in analyzing the cash flows and discount rates of rebalanced portfolios.

We study sources of variation in five well-known anomaly portfolios, the market portfolio, and the mean-variance efficient combination of these factor portfolios. Cash flow shocks to the stocks underlying the MVE portfolio account for 58% of MVE return variance. Discount rate shocks account for only 13% of MVE return variance. Thus, there is highly negative correlation between cash flow and discount rate shocks (-0.58). Long-run cash flow shocks drive this negative correlation. Any model with a cross-section of stocks has implications for the variance decomposition of the MVE portfolio return and thus the SDF that prices all assets. Forcing models to match these empirical moments restricts the shocks that drive investors' marginal utility and behavioral biases. These empirical patterns also hold broadly across individual anomaly long-short portfolios, thus providing guidance for theories of individual anomalies.

Further, cash flow and discount rate shocks to anomalies exhibit little relation with market cash flow and discount rate shocks. In fact, discount rate shocks to the market are slightly negatively correlated with discount rate shocks to the MVE combination of anomaly

portfolios, casting doubt on theories that rely on common variation in the price of risk (or sentiment) across pricing factors. Based on this evidence, the most promising theories of anomalies are those that emphasize the importance of firm-level long-run cash flow shocks as drivers of changes in firm risk or errors in investors' expectations.

We provide preliminary evidence that traditional macroeconomic variables are weakly linked to the cash flows of anomaly portfolios. In future research, we hope to better understand the economic drivers of variation in anomaly cash flows to help explain why anomaly cash flows, unlike industry cash flows, are priced.

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**Table 1 - Summary Statistics**

Table 1: Panel A shows summary statistics for firm-level returns, cash flows, and characteristics. The first column lists the variables as defined in the text. The second column reports the number of firm-year observations,  $N$ . The remaining columns report the mean, standard deviation and various percentiles of the firm-year distribution for each variable. Panel B provides the correlation matrix for the firm accounting characteristics. The sample spans the years 1964 through 2015.

<b>Panel A:</b>	$N$	Mean	SD	P1	P10	P50	P90	P99
AnnRet	68,639	12.66	50.70	-81.06	-39.82	7.69	65.60	175.00
Rf	68,639	5.35	3.12	0.12	0.31	5.55	8.61	13.96
Volat	63,561	38.69	16.94	15.34	21.83	35.11	58.75	102.45
SizeWt	68,639	0.31	1.06	0.00	0.01	0.06	0.55	5.38
lnROE <sup>CS</sup>	65,275	11.29	32.27	-86.64	-11.90	10.23	36.33	117.96
lnBM	67,296	-0.72	0.83	-3.04	-1.78	-0.66	0.29	0.98
lnME	68,590	4.89	1.31	2.90	3.37	4.65	6.73	8.62
lnProf	66,361	21.27	26.92	-72.50	6.39	23.33	39.14	77.92
lnInv	67,475	16.10	28.27	-31.35	-4.87	9.91	42.83	132.75

<b>Panel B:</b>	1	2	3	4
lnBM (1)	1.00			
lnME (2)	-0.28	1.00		
lnProf (3)	-0.11	0.19	1.00	
lnInv (4)	-0.19	-0.03	0.02	1.00
lnMom (5)	-0.25	0.16	0.06	-0.07

**Table 2 - Return and Earnings Forecasting Regressions**

Table 2: The table shows forecasting regressions of firms' log annual real returns ( $\ln\text{Ret}$ ), log annual real clean-surplus earnings ( $\ln\text{ROE}^{\text{CS}}$ ), and log book-to-market ( $\ln\text{BM}$ ) on one-year-lagged value of a set of characteristics:  $\ln\text{BM}$ , log profitability ( $\ln\text{Prof}$ ), log asset growth ( $\ln\text{Inv}$ ), log market equity ( $\ln\text{ME}$ ), log 6-month momentum percentile ( $\ln\text{Mom6}$ ), realized variance ( $\ln\text{RV}$ ), and the log one-year real risk-free rate ( $\ln\text{Rf}$ ). We define interactions between each of the first five characteristics and  $\ln\text{BM}$  (e.g.,  $\text{Ind}_{\text{Inv}} * \ln\text{BM}$ ) as explained in the main text. The sample spans the years 1964 through 2015. Standard errors clustered by year and firm appear in parenthesis. N denotes the number of observations. The marks '+', '\*\*', and '\*\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

	$\ln\text{Ret}$	$\ln\text{ROE}^{\text{CS}}$	$\ln\text{BM}$
Lag $\ln\text{Ret}$	0.011 (0.080)	0.127** (0.025)	0.121+ (0.072)
Lag $\ln\text{ROE}^{\text{CS}}$	-0.036 (0.035)	-0.040* (0.017)	-0.004 (0.029)
Lag $\ln\text{BM}$	0.051* (0.021)	-0.109** (0.009)	0.875** (0.023)
Lag $\ln\text{Prof}$	0.043** (0.014)	0.069** (0.009)	0.0266 (0.018)
Lag $\ln\text{Inv}$	-0.051** (0.014)	0.003 (0.005)	0.057** (0.011)
Lag $\ln\text{ME}$	-0.019 (0.018)	-0.003 (0.005)	0.017+ (0.010)
Lag $\ln\text{Mom6}$	0.019 (0.018)	-0.005 (0.006)	-0.026+ (0.015)
Lag $\ln\text{RV}$	-0.030 (0.021)	-0.006 (0.007)	0.025 (0.019)
Lag $\text{Ind}_{\text{BM}} \times \ln\text{BM}$	0.013 (0.012)	-0.018** (0.005)	0.005 (0.013)
Lag $\text{Ind}_{\text{Prof}} \times \ln\text{BM}$	-0.009 (0.007)	0.022** (0.003)	0.032** (0.007)
Lag $\text{Ind}_{\text{Inv}} \times \ln\text{BM}$	0.004 (0.004)	-0.010** (0.003)	-0.015** (0.005)
Lag $\text{Ind}_{\text{ME}} \times \ln\text{BM}$	-0.007 (0.007)	0.014** (0.004)	0.022** (0.008)
Lag $\text{Ind}_{\text{Mom6}} \times \ln\text{BM}$	0.011+ (0.006)	-0.015** (0.003)	-0.027** (0.008)
Lag $\ln\text{Rf}$	0.002 (0.028)	0.013 (0.008)	0.011 (0.024)
$R^2$	0.045	0.241	0.698
$N$	49,755	49,755	49,755

**Table 3 - Characteristic Forecasting Regressions**

Table 3: Panel A shows annual forecasting regressions of firm characteristics on their own lag as well as the firm’s lagged book-to-market ratio. The characteristics are log profitability (lnProf), log asset growth (lnInv), log market equity (lnME), log three-year issuance (lnIssue), and realized variance (lnRV), as well as interactions with lnBM as explained in the main text. Panel B reports the regression coefficients of the aggregate variable, the log one-year real risk-free rate (lnRf), which is regressed only on its own lag. The sample spans the years 1964 through 2015. Standard errors clustered by year and firm appear in parenthesis. N denotes the number of observations. The marks ‘+’, ‘\*’, and ‘\*\*’ indicate significance at the 10, 5, and 1 percent levels, respectively.

<b>Panel A:</b>	Own Lag	Lag lnBM	$R^2$	$N$
lnProf	0.734** (0.052)	-0.080** (0.018)	46.7%	49,708
lnInv	0.157** (0.026)	-0.301** (0.026)	15.7%	49,720
lnME	0.978** (0.005)	0.022 (0.014)	91.1%	49,749
lnMom6	0.048* (0.019)	0.056* (0.027)	0.3%	49,748
lnRV	0.688** (0.068)	-0.056* (0.035)	48.9%	49,408
Ind <sub>BM</sub> × lnBM	0.566** (0.025)	-0.307** (0.039)	60.7%	49,755
Ind <sub>Prof</sub> × lnBM	0.666** (0.020)	0.118** (0.013)	51.7%	49,755
Ind <sub>Inv</sub> × lnBM	0.258** (0.013)	0.163** (0.018)	11.0%	49,755
Ind <sub>ME</sub> × lnBM	0.685** (0.021)	0.149** (0.017)	54.9%	49,755
Ind <sub>Mom6</sub> × lnBM	0.022 (0.027)	0.157** (0.014)	2.0%	49,755

<b>Panel B:</b>	Own Lag	$R^2$	$N$
Lag lnRf	0.603** (0.200)	36.3%	51

**Table 4 - Variance Decompositions**

Table 4: The table displays the variance decomposition of firm-level and market-level real returns into CF and DR components. "Panel VAR" means that the CF and DR shocks are retrieved from our main panel VAR, while "Time-series VAR", which only is relevant for the market portfolio, refers to a VAR run directly at the market level, using market returns, earnings and book-to-market ratios. surplus earnings, returns, and book-to-market ratio. The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*\*', and '\*\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

	$var(DR)$	$var(CF)$	$var(Cross)$	$-2cov(DR, CF)$	$corr(DR, CF)$	$corr(Pred, Act)$
<b>Firm-level returns</b>						
<i>Panel VAR</i>						
Fraction of $var(r)$	0.225 <sup>+</sup> (0.122)	0.533 <sup>**</sup> (0.115)		0.243 <sup>**</sup> (0.075)	-0.351 <sup>*</sup> (0.162)	
<b>Market returns</b>						
<i>Panel VAR</i>						
Fraction of $var(r_m)$	0.178 (0.119)	0.552 <sup>**</sup> (0.191)	0.008 <sup>**</sup> (0.003)	0.308 <sup>+</sup> (0.213)	-0.492 (0.452)	0.985 <sup>**</sup> (0.002)
<i>Time-series VAR</i>						
Fraction of $var(r_m)$	0.281 (0.236)	0.248 (0.187)		0.471 <sup>**</sup> (0.144)	-0.892 <sup>**</sup> (0.249)	

**Table 5 - Anomaly Variance Decompositions**

Table 5: Panel A of the table shows decompositions of the variance of log anomaly returns into cash flow (CF) and discount rate (DR) components. The anomaly return is the difference between the log return of the top quintile portfolio and the log return of the bottom quintile portfolio, where the quintile sort is based on the relevant characteristic. Panel B shows variance decompositions of log returns to alternative mean-variance efficient (MVE) portfolios: the first is the in-sample MVE portfolio based on the quintile long-short anomaly portfolios only, the second includes also the market portfolio. The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*\*', and '\*\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

	$var(DR)$	$var(CF)$	$var(Cross)$	$-2cov(DR, CF)$	$corr(DR, CF)$	$corr(Pred, Act)$
<b>Panel A: Individual long-short anomaly portfolios</b>						
<i>Book-to-market:</i>						
Fraction of $var(r_{bm})$	0.180 (0.116)	0.536** (0.169)	0.022* (0.011)	0.316** (0.122)	-0.509** (0.195)	0.958** (0.041)
<i>Profitability:</i>						
Fraction of $var(r_{prof})$	0.232+ (0.137)	0.529** (0.165)	0.042+ (0.022)	0.326** (0.128)	-0.466** (0.166)	0.884** (0.072)
<i>Size:</i>						
Fraction of $var(r_{size})$	0.170 (0.121)	0.399* (0.174)	0.028+ (0.017)	0.343** (0.113)	-0.659** (0.154)	0.910** (0.059)
<i>Momentum:</i>						
Fraction of $var(r_{mom})$	0.157 (0.106)	0.436** (0.167)	0.024** (0.010)	0.361** (0.114)	-0.690** (0.143)	0.955** (0.042)
<i>Investment:</i>						
Fraction of $var(r_{inv})$	0.177+ (0.098)	0.563** (0.167)	0.029* (0.015)	0.366** (0.128)	-0.579** (0.163)	0.945** (0.047)
<b>Panel B: MVE portfolios</b>						
<i>MVE portfolio, ex market:</i>						
Fraction of $var(r_{mve}^{ex\ mkt})$	0.142 (0.097)	0.440** (0.173)	0.026** (0.010)	0.373** (0.118)	-0.747** (0.134)	0.924** (0.056)
<i>MVE portfolio, incl. market:</i>						
Fraction of $var(r_{mve}^{all})$	0.131 (0.092)	0.580** (0.182)	0.025* (0.011)	0.322* (0.152)	-0.583+ (0.323)	0.932** (0.053)



**Table 6 - Correlations between Anomaly and Market Return Components**

Table 6: Panel A of the table shows correlations between market cash flow and discount rate shocks and the anomaly cash flow and discount rate shocks. Panel B shows correlations between market cash flow and discount rate shocks and the cash flow and discount rate shocks of the mean-variance efficient (MVE) portfolio, where the latter is constructed as the in-sample MVE portfolio based on the quintile long-short anomaly portfolios *only* – thus, the market portfolio is not included in the MVE portfolio construction. The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*', and '\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

	Market CF		Market DR	
	Anomaly CF	Anomaly DR	Anomaly CF	Anomaly DR
<b>Panel A:</b>				
Book-to-market	-0.12 (0.21)	-0.20 (0.16)	0.38* (0.16)	0.12 (0.21)
Profitability	-0.10 (0.17)	0.20 (0.20)	0.06 (0.21)	-0.45** (0.14)
(-) Investment	-0.06 (0.21)	-0.02 (0.17)	0.35* (0.16)	-0.02 (0.17)
(-) Size	-0.08 (0.19)	-0.36* (0.18)	-0.08 (0.18)	0.27 (0.22)
Momentum	0.08 (0.16)	0.06 (0.18)	-0.14 (0.15)	-0.12 (0.17)
<b>Panel B:</b>				
MVE portfolio, ex. market	-0.01 (0.16)	0.06 (0.20)	0.14 (0.15)	-0.27 (0.17)

**Table 7 - Anomaly Variance Decompositions:  
Short- and Long-run Cash Flow Shocks**

Table 7: The table shows correlations between short-run and long-run cash flow shocks, as well as the correlation between short- and long-run cash flow shocks and discount rate shocks. The anomaly return is the difference between the log return of the top quintile portfolio and the log return of the bottom quintile portfolio, where the quintile sort is based on the relevant characteristic. The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*', and '\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

	$Corr(CF_{short-run}, CF_{long-run})$	$Corr(DR, CF_{short-run})$	$Corr(DR, CF_{long-run})$
<i>Book-to-market:</i>	-0.43** (0.16)	0.11 (0.16)	-0.55** (0.18)
<i>Profitability:</i>	-0.55** (0.12)	-0.16 (0.16)	-0.28 (0.20)
<i>Size:</i>	-0.15 (0.15)	0.17 (0.15)	-0.77** (0.11)
<i>Investment:</i>	-0.32* (0.15)	-0.04 (0.17)	-0.57** (0.14)
<i>Momentum:</i>	-0.49** (0.13)	-0.03 (0.17)	-0.59** (0.18)
<i>Market:</i>	0.09 (0.25)	0.04 (0.32)	-0.58 (0.46)

**Table 8 - Correlations of CF and DR shocks with aggregate metrics**

Table 8: The table shows contemporaneous correlations between the market and anomaly portfolios' cash flow and discount rate shocks and shocks to various aggregate metrics: 1-year log real per capita consumption growth, 1-year real per capita GDP growth, the 1-year difference in the log labor share, 3-year consumption growth (current and future 2 years), one year log difference in Baker and Wurgler's sentiment index, the 1-year difference in the Baa-Aaa yield spread, and the cumulated one year shock to the Broker-Dealer leverage factor of Adrien, Etula, and Muir (2014). A bold number implies significant at the 5 percent level, a number in italics implies significance at the 10 percent level. The sample spans the years 1964 through 2015. We do not take into account, however, estimation error in the shocks arising from estimation error in the VARs when calculating statistical significance.

	1yr GDP growth	1yr Cons. growth	Labor share	3yr Cons. growth	Sentiment	Baa-Aaa spread	B-D Lev. Factor
<b>CF correlations:</b>							
<i>Market:</i>	<b>0.41</b>	<b>0.45</b>	<b>-0.33</b>	<b>0.20</b>	-0.03	<b>-0.38</b>	<b>0.31</b>
<i>Book-to-market:</i>	-0.09	-0.17	0.00	-0.05	0.14	-0.10	<i>0.22</i>
<i>(-) Investment:</i>	<i>-0.20</i>	-0.15	-0.09	-0.05	0.09	-0.02	<i>0.20</i>
<i>Profitability:</i>	0.11	-0.02	0.11	0.08	-0.02	0.03	0.02
<i>(-) Size:</i>	-0.04	0.11	-0.15	<i>0.21</i>	0.01	0.06	<i>0.20</i>
<i>Momentum:</i>	0.19	<i>0.22</i>	0.09	0.15	-0.09	-0.10	0.06
<i>MVE ex market:</i>	0.15	0.11	0.11	0.14	-0.01	-0.15	<i>0.24</i>
<i>MVE all:</i>	<b>0.35</b>	<b>0.32</b>	-0.13	<i>0.20</i>	0.00	<b>-0.43</b>	<b>0.40</b>
<b>DR correlations:</b>							
<i>Market:</i>	-0.02	<i>-0.20</i>	0.11	<i>-0.22</i>	0.13	-0.04	0.09
<i>Book-to-market:</i>	0.03	0.03	<b>0.30</b>	-0.00	-0.03	0.19	-0.01
<i>(-) Investment:</i>	0.05	-0.02	0.04	-0.17	-0.04	0.09	-0.04
<i>Profitability:</i>	0.02	0.18	-0.16	0.08	-0.02	-0.09	<b>-0.32</b>
<i>(-) Size:</i>	-0.12	<b>-0.25</b>	<b>0.40</b>	-0.15	-0.10	0.15	-0.19
<i>Momentum:</i>	<b>-0.25</b>	-0.17	-0.15	0.01	<i>-0.20</i>	0.13	-0.15
<i>MVE ex market:</i>	<i>-0.24</i>	-0.10	-0.10	0.00	<b>-0.25</b>	0.21	<b>-0.33</b>
<i>MVE all:</i>	-0.16	-0.17	-0.17	-0.18	-0.08	0.13	<i>-0.20</i>

**Table 9 - Market Variance Decompositions in Alternative Specifications**

Table 9: The table shows variance decompositions of market log returns into cash flow (CF) and discount rate (DR) components, derived from alternative specifications of the firm-level panel VAR, as explained in the text. Spec1 refers to the simplest VAR specification without interaction terms. Spec2 refers to the specification that includes the aggregate book-to-market ratio. Spec3 refers to the specification that in addition includes interaction terms. Spec4 and Spec5 refer to specifications including industry- and firm-specific rolling means, respectively. The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*', and '\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

		$var(DR)$	$var(CF)$	$var(Cross)$	$-2cov(DR, CF)$	$corr(DR, CF)$	$corr(Pred, Act)$
<i>Spec1</i>	Fraction of $var(r_m)$	0.207 (0.139)	0.642** (0.226)	0.010* (0.005)	0.194 (0.278)	-0.266 (0.466)	0.987** (0.023)
<i>Spec2</i>	Fraction of $var(r_m)$	1.073 (1.076)	0.318 (0.231)	0.008 (0.015)	-0.344 (1.206)	0.294 (0.817)	0.987** (0.023)
<i>Spec3</i>	Fraction of $var(r_m)$	1.498 (2.876)	0.551 (0.842)	0.040 (0.173)	-0.876 (3.389)	0.482 (1.067)	0.987** (0.023)
<i>Spec4</i>	Fraction of $var(r_m)$	0.177 (0.123)	0.550** (0.195)	0.007+ (0.004)	0.182 (0.242)	-0.292 (0.476)	0.988** (0.022)
<i>Spec5</i>	Fraction of $var(r_m)$	0.204 (0.138)	0.559** (0.212)	0.007* (0.003)	0.200 (0.256)	-0.297 (0.473)	0.987** (0.022)

**Table 10 - Anomaly Variance Decompositions in Alternative Specifications**

Table 10: The table shows decompositions of the variance of log long-short anomaly returns into cash flow (CF) and discount rate (DR) components. The variance-decompositions are derived from alternative specifications of the firm-level panel VAR, as explained in the text. Spec1 refers to the simplest VAR specification without interaction terms. Spec2 refers to the specification that includes the aggregate book-to-market ratio. Spec3 refers to the specification that in addition includes interaction terms. Spec4 and Spec5 refer to specifications including industry-and firm-specific rolling means, respectively. The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*', and '\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively.

		<i>var (DR)</i>	<i>var (CF)</i>	<i>var (Cross)</i>	<i>-2cov (DR, CF)</i>	<i>corr (DR, CF)</i>	<i>corr (Pred, Act)</i>
<i>Book-to-market:</i>							
<i>Spec1</i>	Fraction of <i>var (r<sub>bm</sub>)</i>	0.166 <sup>+</sup> (0.100)	0.477** (0.152)	0.017* (0.008)	0.354** (0.121)	-0.628** (0.190)	0.967** (0.040)
<i>Spec2</i>	Fraction of <i>var (r<sub>bm</sub>)</i>	0.087 (0.078)	0.705** (0.272)	0.030 (0.033)	0.253* (0.123)	-0.512* (0.228)	0.968** (0.036)
<i>Spec3</i>	Fraction of <i>var (r<sub>bm</sub>)</i>	0.104 (0.087)	1.140 (0.818)	0.095 (0.251)	-0.040 (0.377)	0.057 (0.517)	0.967** (0.036)
<i>Spec4</i>	Fraction of <i>var (r<sub>bm</sub>)</i>	0.152 <sup>+</sup> (0.090)	0.413** (0.133)	0.015* (0.007)	0.312** (0.114)	-0.624** (0.184)	0.971** (0.034)
<i>Spec5</i>	Fraction of <i>var (r<sub>bm</sub>)</i>	0.171 <sup>+</sup> (0.107)	0.425** (0.155)	0.015* (0.007)	0.350** (0.121)	-0.649** (0.204)	0.967** (0.036)
<i>Profitability:</i>							
<i>Spec1</i>	Fraction of <i>var (r<sub>prof</sub>)</i>	0.179 <sup>+</sup> (0.107)	0.487** (0.154)	0.033** (0.013)	0.360** (0.121)	-0.610** (0.159)	0.892** (0.068)
<i>Spec2</i>	Fraction of <i>var (r<sub>prof</sub>)</i>	0.122 (0.098)	0.663** (0.243)	0.044 (0.038)	0.275* (0.137)	-0.480* (0.214)	0.900** (0.066)
<i>Spec3</i>	Fraction of <i>var (r<sub>prof</sub>)</i>	0.320 (0.223)	0.735* (0.329)	0.060 (0.133)	0.010 (0.323)	-0.010 (0.336)	0.914** (0.061)
<i>Spec4</i>	Fraction of <i>var (r<sub>prof</sub>)</i>	0.177 <sup>+</sup> (0.103)	0.403** (0.133)	0.026** (0.011)	0.328** (0.115)	-0.614** (0.162)	0.914** (0.060)
<i>Spec5</i>	Fraction of <i>var (r<sub>prof</sub>)</i>	0.203 (0.138)	0.430** (0.161)	0.026** (0.010)	0.334** (0.122)	-0.566** (0.203)	0.903** (0.066)
<i>Size:</i>							
<i>Spec1</i>	Fraction of <i>var (r<sub>size</sub>)</i>	0.160 (0.111)	0.386* (0.155)	0.021 (0.010)	0.362** (0.115)	-0.727** (0.171)	0.937** (0.050)
<i>Spec2</i>	Fraction of <i>var (r<sub>size</sub>)</i>	0.080 (0.083)	0.617** (0.283)	0.031 (0.024)	0.279* (0.128)	-0.625** (0.194)	0.937** (0.050)
<i>Spec3</i>	Fraction of <i>var (r<sub>size</sub>)</i>	0.285 (0.379)	0.967 (0.686)	0.081 (0.160)	-0.112 (0.569)	0.107 (0.452)	0.943** (0.047)
<i>Spec4</i>	Fraction of <i>var (r<sub>size</sub>)</i>	0.150 (0.103)	0.340** (0.138)	0.023* (0.010)	0.317** (0.108)	-0.702** (0.172)	0.935** (0.050)
<i>Spec5</i>	Fraction of <i>var (r<sub>size</sub>)</i>	0.141 (0.106)	0.389** (0.167)	0.020* (0.010)	0.327** (0.107)	-0.699** (0.161)	0.903** (0.062)
<i>Investment:</i>							
<i>Spec1</i>	Fraction of <i>var (r<sub>inv</sub>)</i>	0.169 <sup>+</sup> (0.087)	0.530** (0.161)	0.022** (0.007)	0.374** (0.126)	-0.625** (0.166)	0.950** (0.045)
<i>Spec2</i>	Fraction of <i>var (r<sub>inv</sub>)</i>	0.086 (0.066)	0.883* (0.389)	0.054 (0.063)	0.212 (0.179)	-0.386 (0.351)	0.951** (0.044)
<i>Spec3</i>	Fraction of <i>var (r<sub>inv</sub>)</i>	0.087 (0.051)	0.991 (0.872)	0.095 (0.276)	0.142 (0.328)	-0.241 (0.638)	0.953** (0.043)
<i>Spec4</i>	Fraction of <i>var (r<sub>inv</sub>)</i>	0.162 <sup>+</sup> (0.083)	0.425** (0.133)	0.015** (0.005)	0.341** (0.115)	-0.651** (0.169)	0.956** (0.041)
<i>Spec5</i>	Fraction of <i>var (r<sub>inv</sub>)</i>	0.182 <sup>+</sup> (0.100)	0.438** (0.174)	0.019** (0.007)	0.351** (0.121)	-0.620** (0.176)	0.955** (0.042)

**Table 11 - Correlations Between Anomaly and Market Return Components:  
Alternative specification using market CF and DR shocks from aggregate VAR**

Table 11: Panel A of the table shows correlations between market cash flow and discount rate shocks and the anomaly cash flow and discount rate shocks. Panel B shows correlations between market cash flow and discount rate shocks and the cash flow and discount rate shocks of the mean-variance efficient (MVE) portfolio, where the latter is constructed as the in-sample MVE portfolio based on the quintile long-short anomaly portfolios *only* – thus, the market portfolio is not included in the MVE portfolio construction. In this alternative specification the market CF and DR shocks are constructed from a time-series market-level VAR (not aggregated from the panel VAR). The sample spans the years 1964 through 2015. Standard errors appear in parentheses. The marks '+', '\*', and '\*\*' indicate significance at the 10, 5, and 1 percent levels, respectively. Standard errors are, however, not corrected from estimation error in estimation of the shocks from the VARs and therefore statistical significance is overstated.

	Market CF		Market DR	
	Anomaly CF	Anomaly DR	Anomaly CF	Anomaly DR
<b>Panel A:</b>				
Book-to-market	-0.30** (0.11)	-0.10 (0.16)	0.30** (0.10)	0.17 (0.16)
Profitability	-0.06 (0.13)	0.35* (0.17)	0.05 (0.11)	-0.40** (0.16)
(-) Investment	-0.24+ (0.14)	0.04 (0.14)	0.27+ (0.14)	-0.04 (0.14)
(-) Size	0.10 (0.15)	-0.45** (0.14)	-0.06 (0.16)	0.43** (0.14)
Momentum	0.19 (0.12)	-0.01 (0.14)	-0.09 (0.13)	-0.08 (0.14)
<b>Panel B:</b>				
MVE portfolio, ex. market	-0.02 (0.12)	0.11 (0.15)	0.13 (0.12)	-0.20 (0.12)

**Figure 1 - Predicting 10-year Market Earnings and Returns**

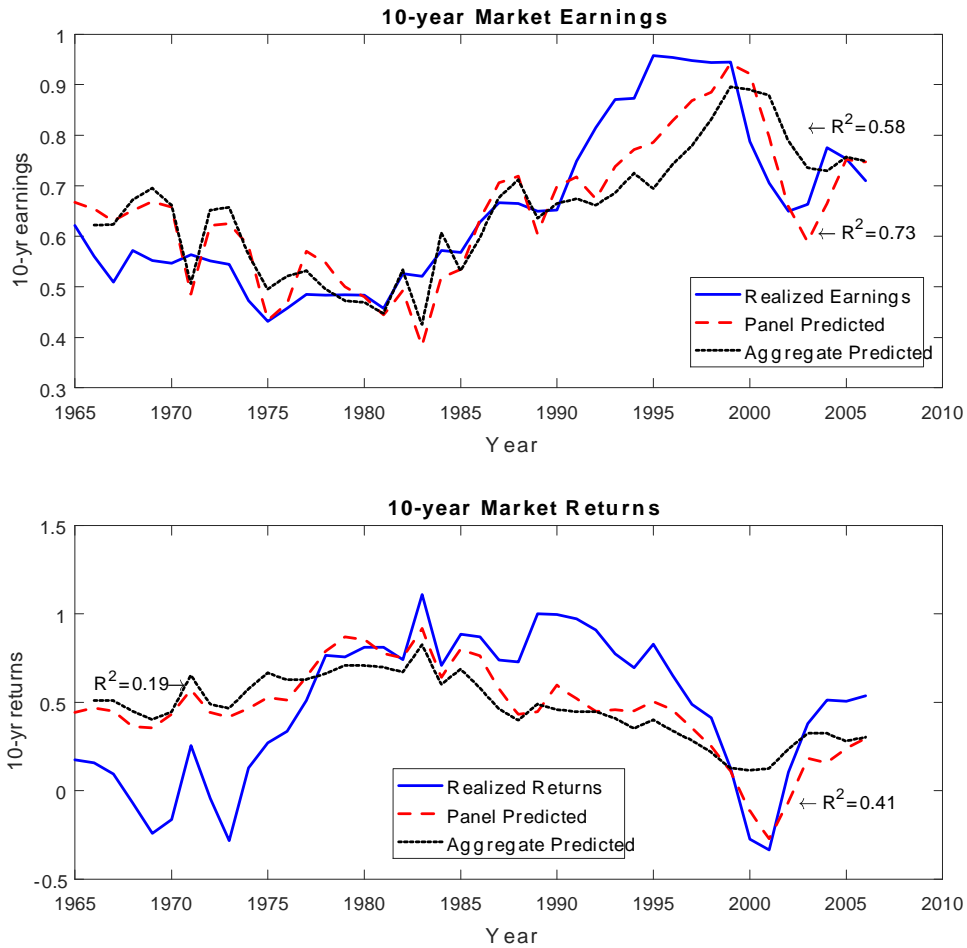


Figure 1: The top plot shows realized versus predicted 10-year log clean surplus earnings of the market portfolio. The solid blue line corresponds to realized earnings, while the dashed red and dotted black lines represent predicted earnings from the panel VAR and market-level VAR, respectively. The year on the x-axis is the year of the prediction—e.g., year 2005 corresponds to the 10-year realized earnings in 2006-2015. The bottom plot shows analogous predictions and realizations of 10-year log real market returns.

**Figure 2 - Predicting 10-year Value Anomaly Earnings and Returns**

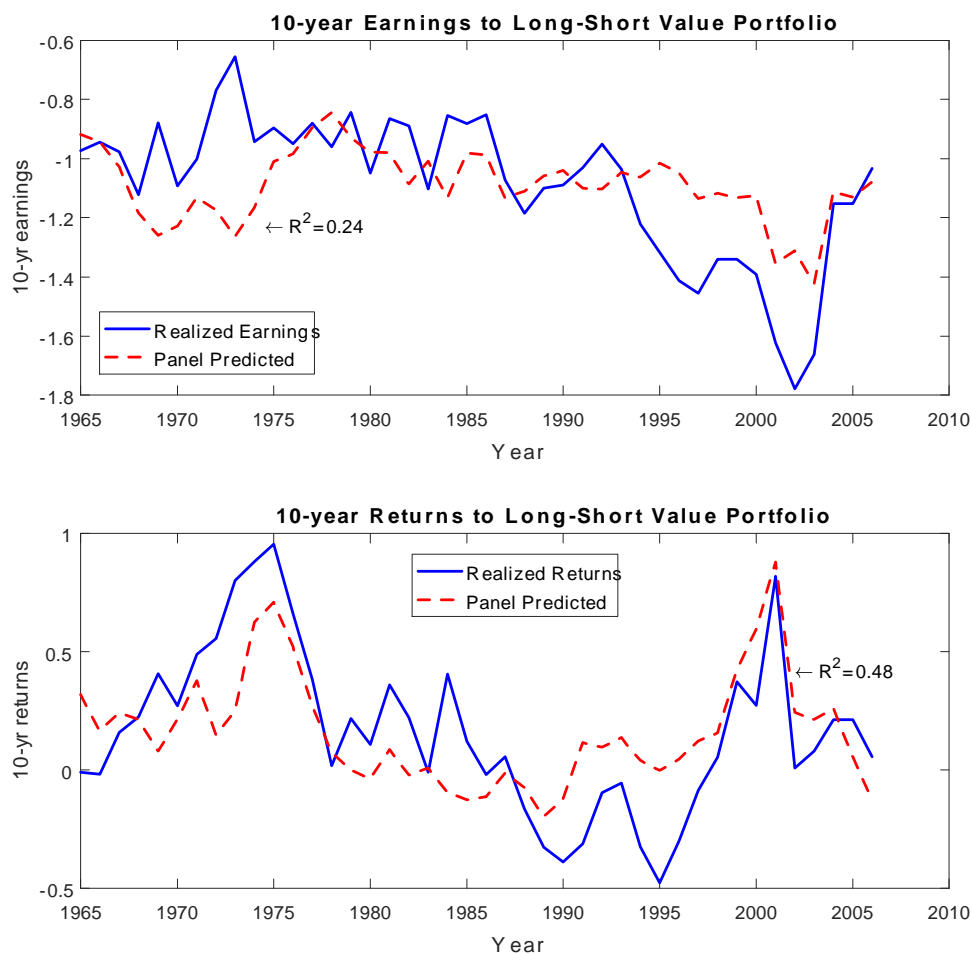


Figure 2: The top plot shows realized versus predicted 10-year log clean-surplus earnings of the long-short portfolio formed by sorting on book-to-market ratios. The solid blue line corresponds to realized earnings, while the dashed red line represent predicted earnings from the panel VAR. The year on the x-axis is the year of the prediction—e.g., year 2005 corresponds to the 10-year realized earnings in 2006-2015. The bottom plot shows the corresponding for 10-year real log returns to the long-short value portfolio.



**Figure 3 - Predictive Power of Valuation Components**

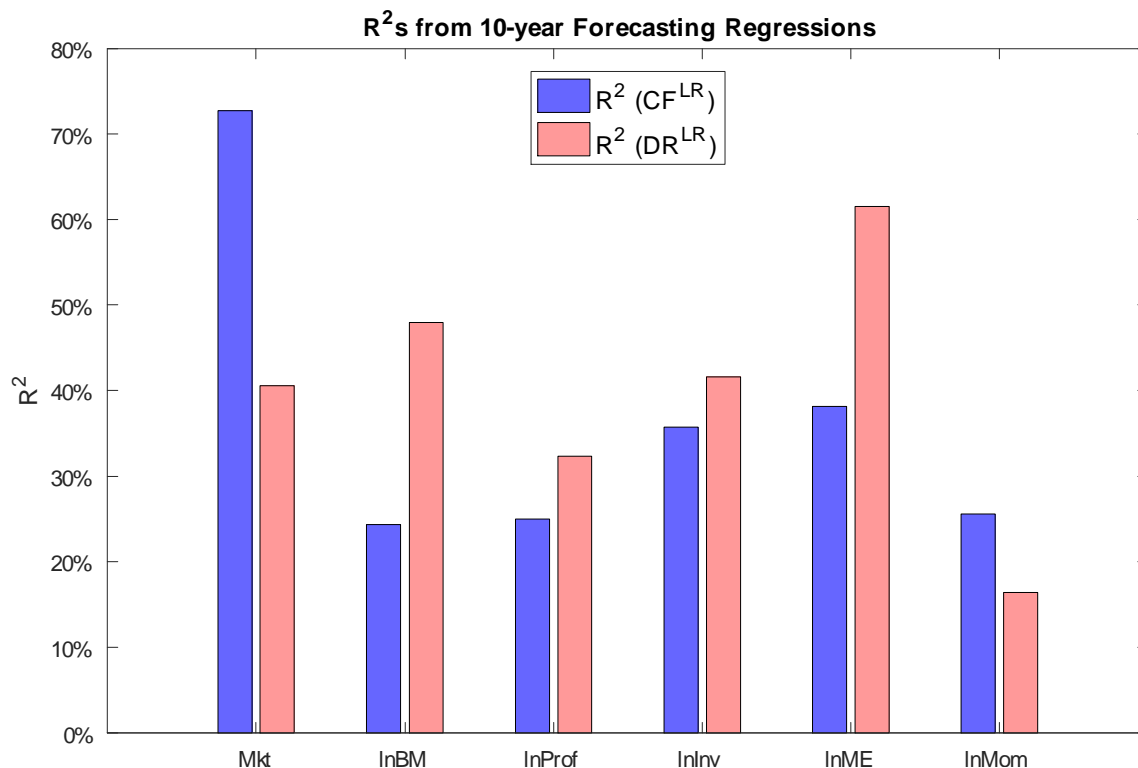


Figure 3: The figure shows  $R^2$  statistics from regressions forecasting either 10-year earnings or returns of portfolios. The blue (left) bars represent the predictive power of regressions of 10-year log clean-surplus earnings on the cash flow components of firms' log book-to-market ratios ( $CF^{LR}$ ) aggregated to the relevant portfolio level. The light red (right) bars represent the predictive power of regressions of 10-year log real returns on the discount rate components of firms' log book-to-market ratios ( $DR^{LR}$ ). The portfolios are the market portfolio (Mkt), as well as top quintile minus bottom quintile portfolios sorted on book-to-market (B/M), profitability (Prof), investment (Inv), size (ME), and issuance (Issue). See the main text for details regarding the construction of the test portfolios and the corresponding cash flow and discount rate components. The sample spans the years 1964 through 2015.

Figure 4 - Market CF and DR shocks vs. anomaly MVE CF and DR shocks

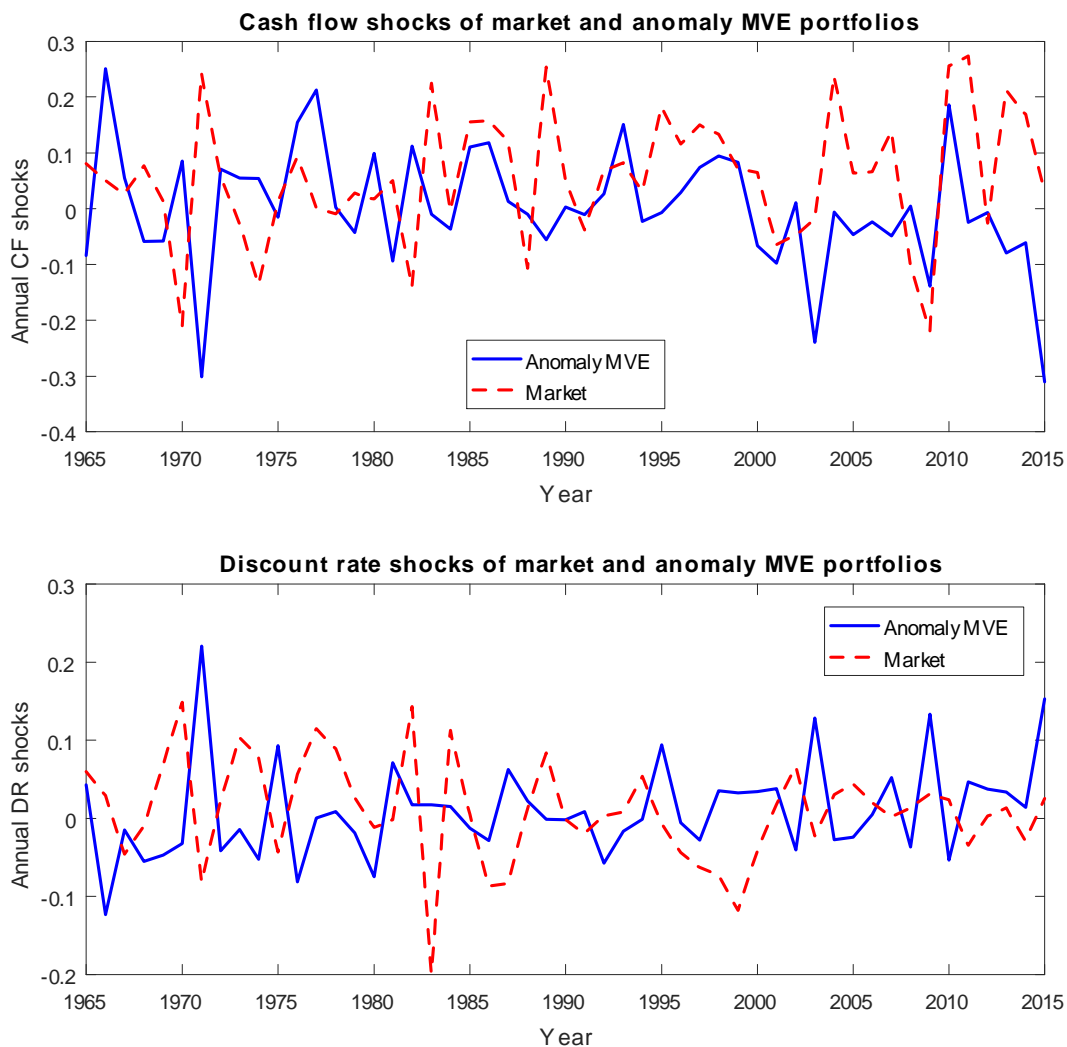


Figure 4: Panel A shows the cash flow shocks from the market and the anomaly mean-variance efficient (MVE) portfolio. The latter is constructed using only the long-short anomaly portfolios and in-sample MVE weights. Panel B shows the same for discount rate shocks. The sample is annual, from 1965 through 2015.

**Figure 5 - Predicting 10-year Market Earnings and Returns in Alternative Specifications**

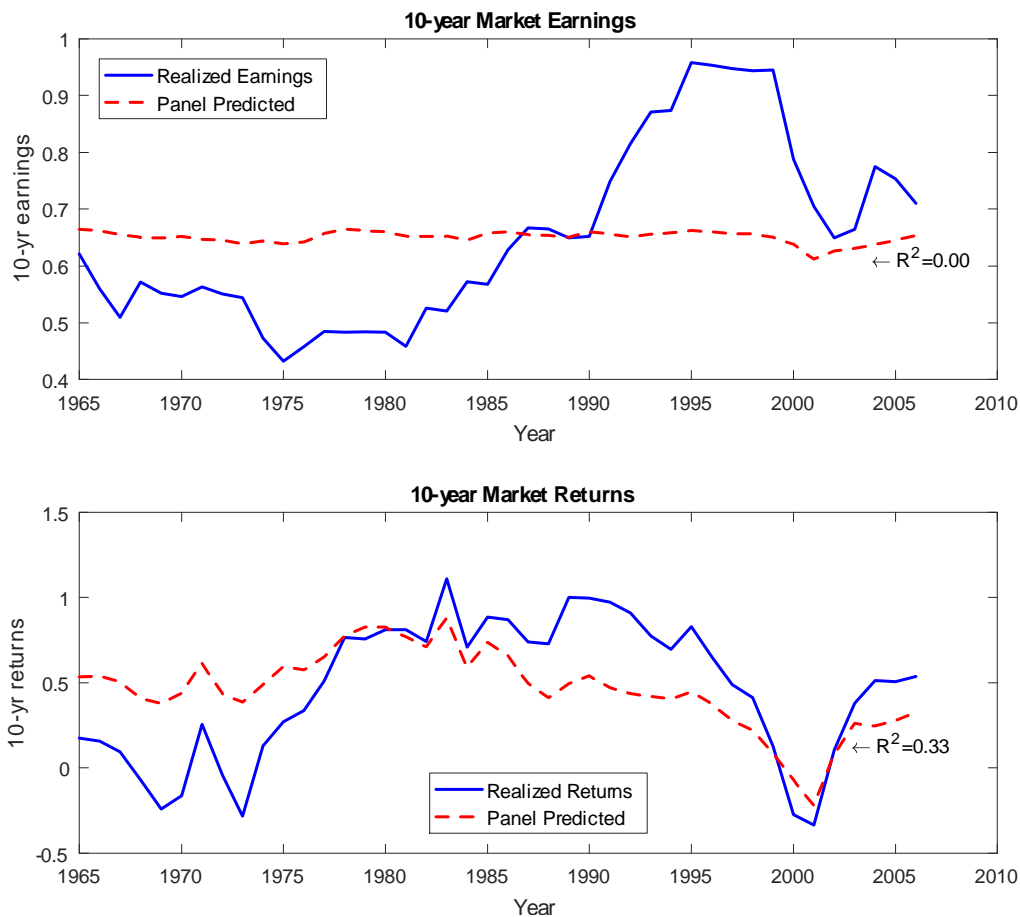


Figure 5: The top plot shows realized versus predicted 10-year log clean surplus earnings of the market portfolio. The solid blue line corresponds to realized earnings, while the red, dashed line represent predicted earnings from an alternative specification of the panel VAR (Spec2, where the aggregate book-to-market ratio is included in the VAR). The year on the x-axis is the year of the prediction—e.g., year 2005 corresponds to the 10-year realized earnings in 2006-2015. The bottom plot shows the corresponding for 10-year log real market returns.

**Figure 6 - Predictive Power of Valuation Components in Alternative Specifications**

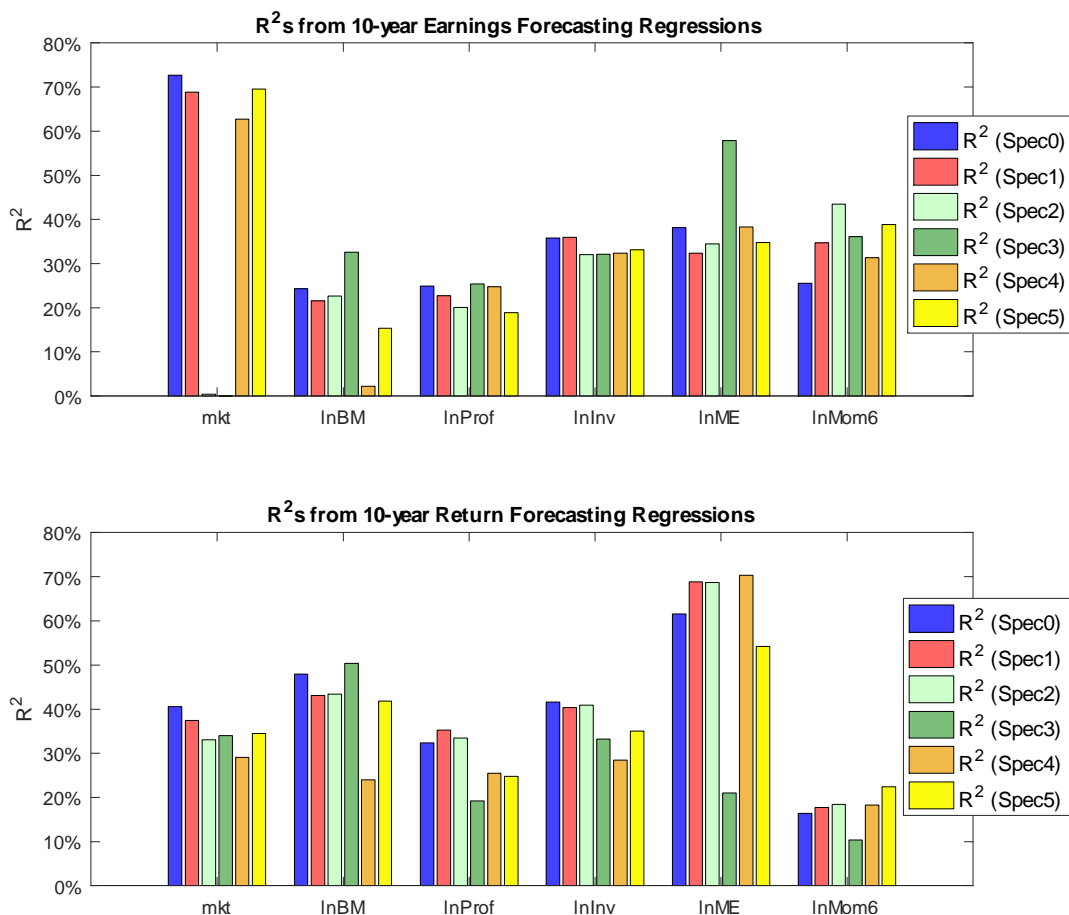


Figure 6: The figure shows the  $R^2$  statistics from regressions forecasting either 10-year earnings (top plot) or returns (bottom plot) of portfolios. The dark blue (left) bars represent the predictive power of regressions using long-run cash flow or discount rate components of the log book-to-market ratios from the main panel VAR specification. The red bars correspond to specification Spec1 (simplest VAR, without interaction terms), the light green bars correspond to specification Spec2 (adding the aggregate book-to-market ratio to the panel VAR). The light brown bars correspond to specification Spec3 (adding both aggregate lnBM and interaction terms, as explained in the text). Finally, the yellow, rightmost bars correspond to specification Spec4 (adding look-ahead-bias-free industry fixed effects). The portfolios are the market portfolio (Mkt), as well as top quintile minus bottom quintile portfolios sorted on book-to-market (B/M), profitability (Prof), investment (Inv), size (ME), and 6-month Momentum (Mom6). See the main text for details regarding the construction of the test portfolios and the corresponding cash flow and discount rate components. The sample spans the years 1964 through 2015.

# Internet Appendix for "What Drives Anomaly Returns?"

by

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## Appendix A: Cash Flows vs. Discount Rates of Trading Strategies

Here we show that the cash flows and discount rates of rebalanced portfolios, such as anomaly portfolios, can differ substantially from those of the underlying firms in the portfolios. We provide examples below in which firms have constant cash flows, but all variation in returns to the rebalanced portfolio comes from cash flow shocks.

We first consider a stylized behavioral model of stock returns and cash flows. Assume that all firms pay constant dividends:

$$D_{i,t} = \bar{d}. \tag{25}$$

Assume that investors in each period erroneously believe that any given firm's dividend is permanently either  $d_L < \bar{d}$  or  $d_H > \bar{d}$ . We define the firms associated with low (high) dividend beliefs to be value (growth) firms. The pricing of these firms satisfies:

$$P^{\text{value}} = \frac{d_L}{R - 1}, \tag{26}$$

$$P^{\text{growth}} = \frac{d_H}{R - 1}, \tag{27}$$

where  $R$  is the gross risk-free rate. Each period, with probability  $q$ , investors switch their beliefs about each stock's dividends either from  $d_L$  to  $d_H$  or from  $d_H$  to  $d_L$ . Investors believe their beliefs will last forever, whereas in reality they will switch with probability  $q$  in each period.

Now consider a value fund that invests only in stocks that investors currently believe will pay dividends of  $d_L$ . Further assume that there are only two firms in the economy—a firm that currently is a growth firm and a firm that currently is a value firm. When beliefs switch, the growth firm becomes a value firm and vice versa. This switch therefore induces trading in the value fund as the fund has to sell firms that become growth firms and buy the new value firms.

Such trading has a significant impact on the fund's dividends. Suppose that the fund

initially holds one share of the value stock, which implies that its initial wealth is  $W_0 = P^{\text{value}}$ . Assume investors do not switch beliefs in the next period. In this case, the fund's gross return is:

$$\begin{aligned} R_1^{\text{value}} &= \frac{P^{\text{value}} + \bar{d}}{P^{\text{value}}} \\ &= 1 + \frac{\bar{d}}{P^{\text{value}}}. \end{aligned} \quad (28)$$

Period 1 cum-dividend wealth is

$$W_1^{\text{cum}} = P^{\text{value}} + \bar{d}, \quad (29)$$

where ex-dividend wealth is  $P^{\text{value}}$  and dividend is  $d_1 = \bar{d}$ . Assume that beliefs switch in period 2. Then:

$$\begin{aligned} R_2^{\text{value}} &= \frac{(R-1) \left( \frac{d_H}{R-1} + \bar{d} \right)}{d_L} \\ &= \frac{d_H + (R-1)\bar{d}}{d_L} \\ &= \frac{d_H}{d_L} + \frac{\bar{d}}{P^{\text{value}}}. \end{aligned} \quad (30)$$

So fund wealth becomes:

$$W_2^{\text{cum}} = P^{\text{value}} \frac{d_H}{d_L} + \bar{d}.$$

Ex-dividend wealth is  $W_2^{\text{ex}} = P^{\text{value}} \frac{d_H}{d_L}$ , and the dividend is  $d_2 = \bar{d}$  once again. The dividend price ratio of the strategy is now:

$$\frac{W_2^{\text{ex}}}{d_2} = \frac{P^{\text{value}}}{\bar{d}} \frac{d_H}{d_L} > \frac{P^{\text{value}}}{\bar{d}}. \quad (31)$$

The higher price-dividend ratio reflects high expected dividend growth next period.

Importantly, the fund now reinvests its capital gain into the current value stock and is able to purchase more than one share. Assuming beliefs do not switch in period 3, the fund's

wealth increases to:

$$\begin{aligned}
W_3^{cum} &= P^{\text{value}} \frac{d_H}{d_L} \left( 1 + \frac{\bar{d}}{P^{\text{value}}} \right) \\
&= P^{\text{value}} \frac{d_H}{d_L} + \frac{d_H}{d_L} \bar{d}.
\end{aligned} \tag{32}$$

Now ex-dividend wealth is  $W_3^{ex} = P^{\text{value}} \frac{d_H}{d_L}$  and  $d_3 = \bar{d} \times d_H/d_L$ , implying that dividend growth during this period is high as  $d_3/d_2 = d_H/d_L > d_2/d_1 = 1$ . The price-dividend ratio of the strategy is now:

$$\frac{W_3^{ex}}{d_3} = \frac{P^{\text{value}} \frac{d_H}{d_L}}{\bar{d} \times d_H/d_L} = \frac{P^{\text{value}}}{\bar{d}}, \tag{33}$$

meaning that the price-dividend ratio returns to its original value.

In summary, dividend growth of the dynamic value strategy varies over time, but expected returns to the strategy are constant and given by:

$$\begin{aligned}
E(R^{\text{value}}) &= 1 - q + q \frac{d_H}{d_L} + \frac{\bar{d}}{P^{\text{value}}} \\
&= 1 - q + q \frac{d_H}{d_L} + \frac{\bar{d}}{d_L} (R - 1).
\end{aligned} \tag{34}$$

A symmetric argument applies to the analogous growth strategy, which also has time-varying dividend growth and constant expected returns. We conclude that return variation in the dynamic trading strategies arises solely because of cash flow shocks even though all firms in the economy incur only discount rate shocks. Firm-level return variation is driven by changes in firms' expected returns, not their dividends—which are constant.

There are no discount rate shocks to the returns of these dynamic strategies when viewed from the perspective of an investor who invests in the value or growth funds. However, unexpected returns to such funds are in fact, under the objective measure of the econometrician, due to discount rate shocks to the underlying firms. The firms' actual expected returns vary, whereas their dividend growth does not.

This feature of rebalanced portfolios is not limited to the case of time-varying mispricing.



Consider a rational model in which value firms have riskier cash flows than growth firms. If time-variation in a firm's cash flow risk causes it to switch between being a value firm and a growth firm, the rational model delivers the same insights as the behavioral model discussed above.

In this example, we assume firms' log dividend growth is:

$$\Delta d_{i,t+1} = -\frac{1}{2}\sigma^2 + \sigma \left( \rho_{s_{i,t}} \varepsilon_{m,t+1} + \sqrt{1 - \rho_{s_{i,t}}^2} \varepsilon_{i,t+1} \right), \quad (35)$$

where  $\varepsilon_{m,t+1}$  and  $\{\varepsilon_{i,t+1}\}_i$  are uncorrelated standard normally distributed shocks representing aggregate and firm-specific dividend shocks, respectively. Firm exposure to aggregate dividend shocks is:

$$\rho_{s_{i,t}} = \begin{cases} \rho^H & \text{if } s_{i,t} = 1 \\ \rho^L & \text{if } s_{i,t} = 0 \end{cases}, \quad (36)$$

where  $s_{i,t}$  follows a two-state Markov process where  $\Pr \{s_{i,t+1} = 1 | s_{i,t} = 0\} = \Pr \{s_{i,t+1} = 0 | s_{i,t} = 1\} = \pi$ . For ease of exposition, set  $\rho^L = 0$  and  $\rho^H = 1$ . Initially, half of firms are in state 1, while the other half are in state 0. If a regime change occurs, all firms currently in state 1 switch to state 0, and vice versa.

The log stochastic discount factor is:

$$m_{t+1} = -\frac{1}{2}\gamma^2\sigma^2 - \gamma\sigma\varepsilon_{m,t+1}, \quad (37)$$

where we implicitly assume a zero risk-free rate and where  $\gamma > 0$  represents risk aversion. These assumptions imply that the conditional mean and volatility of cash flow growth is constant. However, firm risk varies with  $s_{i,t}$ , which determines the covariance of cash flows with the pricing kernel, causing time-varying firm risk premiums.

Solving for the price-dividend ratio as a function of the state yields:

$$\begin{aligned} PD(s_{i,t}) &= E_t \left[ e^{-\frac{1}{2}\gamma^2\sigma^2 - \gamma\sigma\varepsilon_{m,t+1} - \frac{1}{2}\sigma^2 + \sigma \left( \rho_{s_{i,t}} \varepsilon_{m,t+1} + \sqrt{1 - \rho_{s_{i,t}}^2} \varepsilon_{i,t+1} \right)} (1 + PD(s_{i,t+1})) \right] \\ &= e^{-\gamma\sigma^2\rho_{s_{i,t}}} (1 + \pi PD(s_{i,t+1} \neq s_{i,t}) + (1 - \pi) PD(s_{i,t+1} = s_{i,t})). \end{aligned} \quad (38)$$

Denote the price-dividend ratio in state  $j$  as  $PD_j$ . The price-dividend relation above is a system with two equations and two unknowns with the solution:

$$PD_1 = \frac{1}{e^{\gamma\sigma^2} - 1} \quad (39)$$

$$PD_0 = 1/\pi + \frac{1}{e^{\gamma\sigma^2} - 1} \quad (40)$$

These equations show that price-dividend ratios are higher in state 0 when dividend risk is low than in state 1 when dividend risk is high, implying that expected returns are higher in state 0 as expected dividend growth is constant across states. Firms' expected net returns are:

$$E_t [R_{i,t+1}|s_{i,t} = 0] - 1 = 0, \quad (41)$$

$$E_t [R_{i,t+1}|s_{i,t} = 1] - 1 = 2(e^{\gamma\sigma^2} - 1). \quad (42)$$

Since  $PD_0 > PD_1$ , we see that  $E_t [R_{i,t+1}|s_{i,t} = 1] > E_t [R_{i,t+1}|s_{i,t} = 0]$ . Thus, firms' price-dividend ratios fluctuate because of shocks to discount rates, not cash flows. Although there are cash flow shocks in returns arising from the contemporaneous dividend shock ( $\sigma \left( \rho_{s_{i,t}} \varepsilon_{m,t+1} + \sqrt{1 - \rho_{s_{i,t}}^2} \varepsilon_{i,t+1} \right)$ ), dividends are unpredictable and therefore do not induce time-variation in the price-dividend ratio.

Now consider a value mutual fund that in each period buys firms that are currently in the low valuation state 1. With probability  $\pi$ , value firms held by the fund will switch to the high valuation state 0, meaning that they become growth firms. The fund sells all firms in each period and reinvests the proceeds in firms that are in the low valuation state 1. The fund pays out all firm dividends as they occur. The expected return to this strategy is constant and equal to  $E_t [R_{i,t+1}|s_{i,t} = 1] - 1 = 2(e^{\gamma\sigma^2} - 1)$ , even though all firms' expected returns vary over time.

We now analyze the growth of the value fund's dividends in each period. The first source

of fund dividend growth is growth in the underlying firms' dividends, which satisfy:

$$\frac{D_{i,t+1}}{D_{i,t}} = e^{-\frac{1}{2}\sigma^2 + \sigma\varepsilon_{m,t+1}}. \quad (43)$$

The second source of fund dividend growth is growth in the number of shares of value firms held by the fund. If value firms switch to growth firms, the fund will reap a capital gain and be able to buy more shares of the new value firms in the following period. Define the indicator variable  $\mathbf{1}_{s_{i,t} \neq s_{i,t-1}}$  as equal to 1 if there was a regime shift from period  $t - 1$  to period  $t$  and 0 otherwise. Accounting for both sources of growth, fund dividend growth is:

$$\frac{D_{t+1}^{Fund}}{D_t^{Fund}} = \mathbf{1}_{s_{i,t} \neq s_{i,t-1}} \frac{PD_0}{PD_1} e^{-\frac{1}{2}\sigma^2 + \sigma\varepsilon_{m,t+1}} + (1 - \mathbf{1}_{s_{i,t} \neq s_{i,t-1}}) e^{-\frac{1}{2}\sigma^2 + \sigma\varepsilon_{m,t+1}}, \quad (44)$$

where the term  $\frac{PD_0}{PD_1} = 1 + \frac{e^{\gamma\sigma^2} - 1}{\pi}$  represents the capital gain from the prior period. Dividends are predictably high after high capital gains and low after low capital gains. The predictability in dividend growth leads to a time-varying price-dividend ratio for the mutual fund, even though its expected return is constant. Thus, discount rate shocks to the underlying value firms are cash flow shocks for the mutual fund implementing a value trading strategy.

## Appendix B: Relation to Equilibrium Models

The VAR offers a parsimonious, reduced-form model of the cross-section of expected cash flows and discount rates at all horizons. Here we demonstrate that the VAR specification is related to standard asset pricing models. In well-known models such as Campbell and Cochrane's (1999) habit formation model and Bansal and Yaron's (2004) long-run risk model, the log stochastic discount factor is conditionally normally distributed and satisfies:

$$m_{t+1} = -r_{f,t} - \frac{1}{2} \|\lambda_t\|^2 + \lambda_t' \eta_{t+1}, \quad (45)$$

where  $\lambda_t$  is a  $K \times 1$  vector of conditional risk prices,  $\eta_{t+1}$  is a  $K \times 1$  vector of standard normal shocks, and  $r_{f,t}$  is the risk-free rate. With conditionally normal log returns, applying the Law of One Price yields the following expression for the conditional expected log return of firm  $i$ :

$$\begin{aligned} E_t[r_{i,t+1}] &= r_{f,t} - \frac{1}{2} v_{i,t} + cov_t(m_{t+1}, r_{i,t+1}) \\ &= r_{f,t} - \frac{1}{2} v_{i,t} + \beta_{i,t}' \lambda_t, \end{aligned} \quad (46)$$

where  $v_{i,t} \equiv var_t(r_{i,t+1})$  is firm return variance, and  $\beta_{i,t}^{(k)} = \frac{cov_t(\lambda_t^{(k)} \eta_{t+1}^{(k)}, r_{i,t+1})}{var_t(\lambda_t^{(k)} \eta_{t+1}^{(k)})}$  and  $\beta_{i,t} = [\beta_{i,t}^{(1)} \ \beta_{i,t}^{(2)} \ \dots \ \beta_{i,t}^{(K)}]'$  represent firm betas.

We make simplifying assumptions to relate this setup to the VAR specification. Define firm risk premiums as  $z_{i,t}^{(k)} \equiv \beta_{i,t}^{(k)} \lambda_t^{(k)}$  and  $z_{i,t} = [z_{i,t}^{(1)} \ z_{i,t}^{(2)} \ \dots \ z_{i,t}^{(K)}]'$ . Suppose that risk premiums, variances, and the risk-free rate evolve according to:

$$z_{i,t+1} = \bar{z} + A_z (z_{i,t} - \bar{z}) + \Sigma_{z,t} \varepsilon_{i,t+1}^z, \quad (47)$$

$$v_{i,t+1} = \bar{v} + A_v (v_{i,t} - \bar{v}) + \sigma_{v,t} \varepsilon_{i,t+1}^v, \quad (48)$$

$$r_{f,t+1} = \bar{r}_f + A_{r_f} (r_{f,t} - \bar{r}_f) + \sigma_{r,t} \varepsilon_{t+1}^{r_f}, \quad (49)$$

for all firms  $i$ . Assume firm log return on equity is also conditionally normal:

$$e_{i,t+1} = \mu + x_{i,t} + \sigma_{e,t} \varepsilon_{i,t+1}^e, \quad (50)$$

$$x_{i,t+1} = A_x x_{i,t} + \Sigma_{x,t} \varepsilon_{i,t+1}^x, \quad (51)$$

where  $x_{i,t}$  is an  $L \times 1$  vector of latent state variables determining expected return on equity. All shocks can be correlated.

Assuming the clean-surplus model described earlier, firm book-to-market ratios are given by:

$$bm_{i,t} = a_0 + a'_1 r_{f,t} + a'_2 z_{i,t} + a'_3 x_{i,t} + a_4 v_{i,t}. \quad (52)$$

Define the  $(2K + L + 1) \times 1$  vector  $s_{i,t} = [r'_{f,t} \ z'_{i,t} \ v_{i,t} \dots x'_{i,t}]'$  to consist of the stacked state variables. We assume there exist  $(2K + L + 1)$  observed characteristics,  $\xi_{i,t}$ , that span  $s_{i,t}$ :

$$\xi_{i,t} = A_1 + A_2 s_{i,t}, \quad (53)$$

where  $A_2$  is invertible. With the characteristic spanning assumption, firms' book-to-market become a function of the observed characteristics, resulting in a VAR representation of the present-value relation. In sum, the VAR specification concisely summarizes the dynamics of expected cash flows and discount rates, even when both consist of multiple components fluctuating at different frequencies. The VAR yields consistent estimates even though there is heteroskedasticity across firms and time.

When analyzing long-short portfolios, we obtain the anomaly cash flow (discount rate) shock as the difference in the cash flow (discount rate) shocks between the long and short portfolios. Taking the value anomaly as an example, suppose the long value portfolio and short growth portfolio have the same betas with respect to all risk factors except the value factor (say,  $\lambda_t^{(2)}$ ). According to Equation (46), discount rate shocks to this long-short portfolio can only arise from three sources: 1) shocks to the spread in the factor exposure between value and growth firms ( $\beta_{\text{value},t}^{(2)} - \beta_{\text{growth},t}^{(2)}$ ); 2) shocks to the price of risk of the value factor

$(\lambda_t^{(2)})$ ; or 3) shocks to the difference in return variance between the two portfolios. The third possibility arises because we analyze log returns. Similarly, cash flow shocks to this long-short portfolio only reflect these portfolios' differential exposure to cash flow factors.

## Appendix C: Further VAR analysis. Cumulative expected earnings and return responses

We now translate the VAR coefficients into estimates of cumulative expected returns and cash flows at horizons ( $N$ ) ranging from 1 to 20 years. We compute the cumulative coefficients for predicting log returns by summing expected log returns across horizons, discounting by  $\kappa$ , enabling us to express the  $N$ -year discount rate component ( $\widetilde{DR}_{i,t}^{(N)}$ ) as:

$$\widetilde{DR}_{i,t}^{(N)} = E_t \sum_{j=1}^N \kappa^{j-1} \tilde{r}_{i,t+j}, \quad (54)$$

where a tilde above a variable refers to its demeaned value. We plot these cumulative expected returns, which are akin to expected price impacts, for a one-standard deviation increase in each characteristic in Figure A.1. Similarly, Figure A.2 plots the cumulative coefficients for predicting log earnings at horizons from 1 to 20 years. We obtain the  $N$ -year cash flow component of valuations ( $\widetilde{CF}_{i,t}^{(N)}$ ) from the equation:

$$\widetilde{CF}_{i,t}^{(N)} = \sum_{j=1}^N \kappa^{j-1} E_t \left[ \ln \widetilde{ROE}_{i,t+j}^{CS} \right]. \quad (55)$$

These cumulative coefficients allow us to represent the discount rate and cash flow level components in log book-to-market ratios from years 1 through 20 as affine functions of the characteristics in year 0.

Figure A.1 shows that book-to-market and size are the most important predictors of long-run discount rates. The 20-year coefficient on log book-to-market is 26%, while the coefficient on log size is  $-18\%$ . The high persistence of both variables implies that their long-run impacts on valuation are much larger than their short-run impacts. In contrast, some effective predictors of short-run returns, such as log investment, have little long-run impact mainly because they are not very persistent. In addition, investment positively predicts book-to-market ratios, which limits the extent to which its long-run impact can be negative. The long-run value and size coefficients imply that investors heavily discount

the cash flows of value firms, whereas they pay more for the cash flows of large firms. Other notable predictors of 20-year cumulative log returns include log firm profitability and realized variance, which have coefficients of 17% and  $-7\%$ , respectively. The negative effect of realized variance could arise because of the difference between expected log returns and log expected returns or because realized variance negatively forecasts returns as found in Ang et al. (2006).

Figure A.2 shows that book-to-market and size are also the most important predictors of long-run cash flows. The coefficients on log book-to-market and log size are  $-58\%$  and  $-14\%$ , respectively, for predicting cumulative log earnings at the 20-year horizon. In addition, profitability has a 20-year cumulative effect of 17%. These findings indicate that CF and DR shocks are largely driven by shocks to the three most persistent predictive characteristics:  $\ln BM$ ,  $\ln ME$ , and  $\ln Prof$ .



Figure A.1 - Cumulative Return Forecasting Coefficients

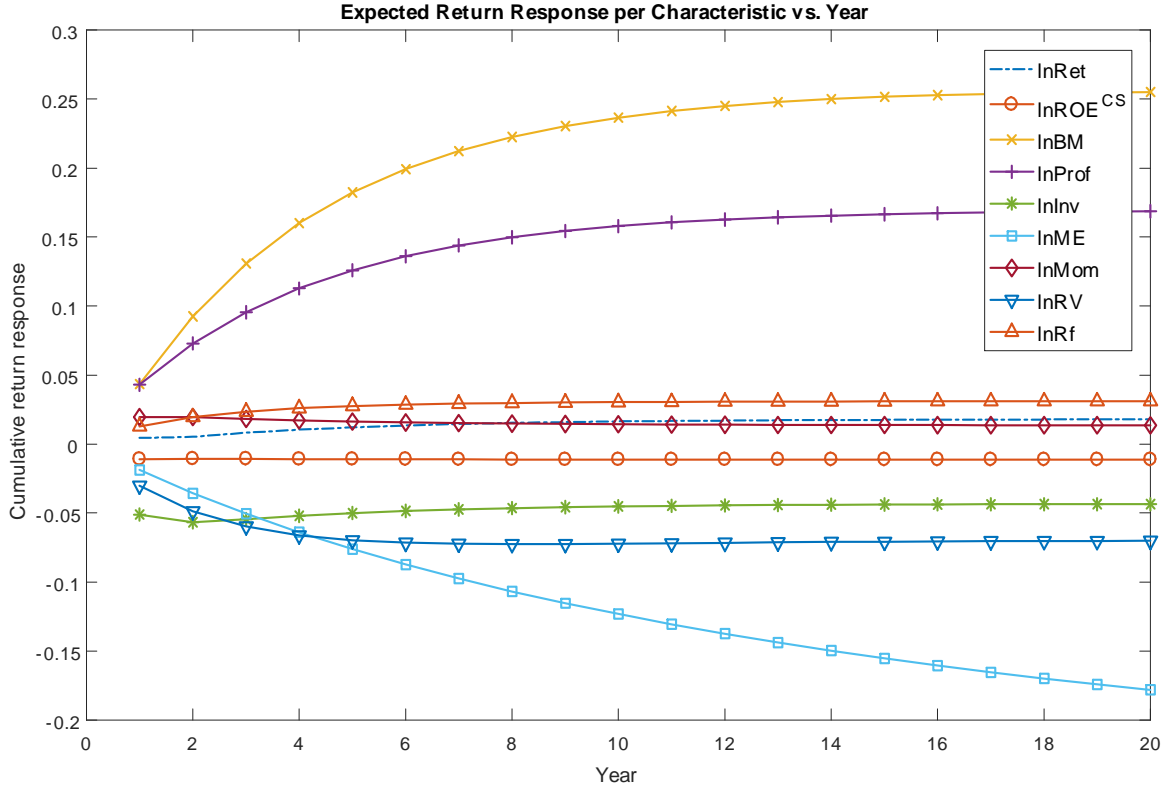


Figure A.1: The plot shows the cumulative coefficients of real log return forecasts based on each characteristic (y-axis) for forecasting horizons of 1 to 20 years (x-axis), as implied by the panel VAR. When computing the cumulative coefficient, the coefficient for horizon  $j$  is multiplied by  $\kappa^j$ , where  $\kappa = 0.96$  as in the text. Thus, the cumulative coefficient for each horizon represents the discount rate component of log book-to-market ratio for that horizon.

Figure A.2 - Cumulative Earnings Forecasting Coefficients

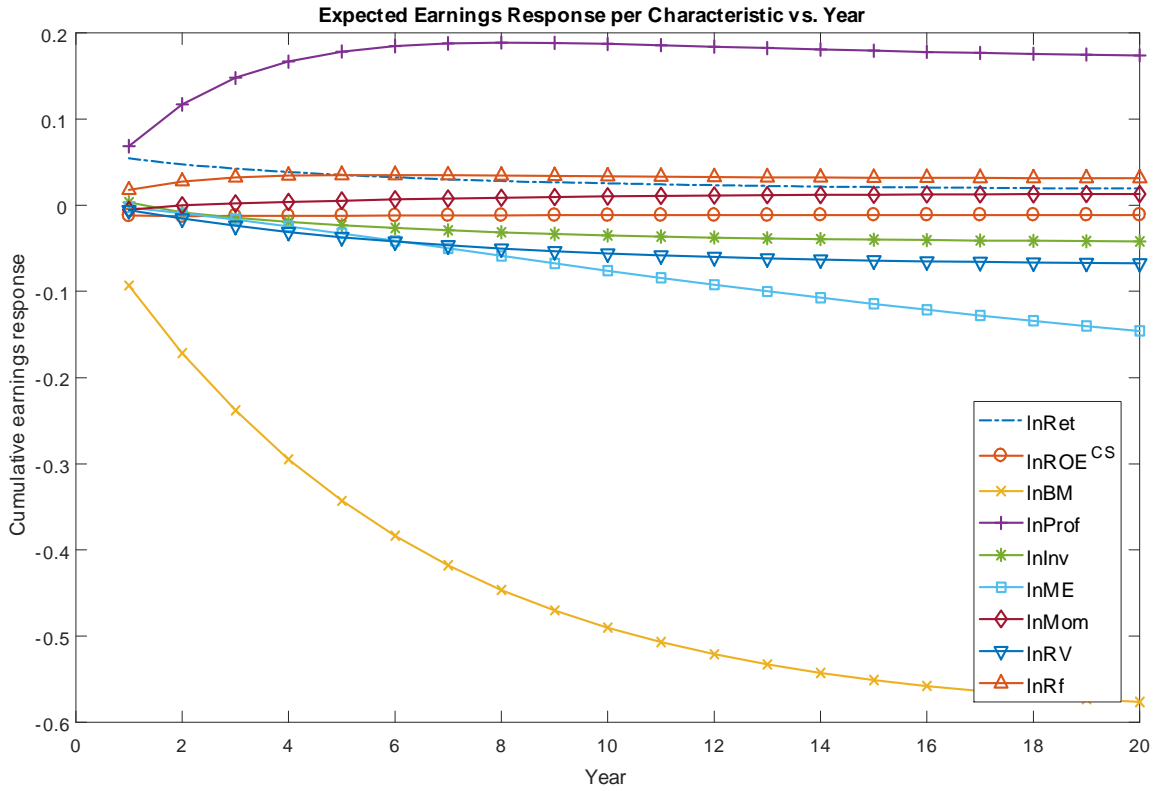


Figure A.2: The plot shows the cumulative coefficients of real log earnings forecasts based on each characteristic (y-axis) for forecasting horizons of 1 to 20 years (x-axis), as implied by the panel VAR. When computing the cumulative coefficient, the coefficient for horizon  $j$  is multiplied by  $\kappa^j$ , where  $\kappa = 0.96$  as in the text. Thus, the cumulative coefficient for each horizon represents the cash flow component of log book-to-market ratio for that horizon.